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Paradoxes and probable truth

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ABSTRACT. The probabilistic account of truth is frequently used to solve paradoxes, nowadays. The article shows it works well in underdetermined cases, and is especially useful to support *non adjunctive* or *truth value gap* strategies. In probabilistic perspective, the *preface paradox* and other similar cases (involving a contrast between distributive and conjunctive valuations), reveal being not really paradoxical, as *non-adjunctive* strategies are perfectly justified. The probabilistic approach also reveals that *truth value gluts* in some contexts are in fact *truth value gaps*: this typically happens in conflicts between epistemic *sources*, or *pieces of evidence*, like in *the reliabilist paradox*, or *Fermi-Hart paradox*. However, the same approach does not really work for *antinomies* (semantic and set-theoretic paradoxes, conveying the schema $\mu \leftrightarrow \neg\mu$), as they does not involve any kind of ignorance or underdetermination.

1. Preface paradox and non-adjunctive strategies

The probabilistic account of truth is frequently used to solve paradoxes.[1] I present here some examples, and I show the probabilistic approach works well when dealing with *epistemic* paradoxes. More specifically: it gives support to *non-adjunctive* and *truth value gap* strategies. But it does not work for *logical*

[1] By 'solving a paradox' I mean 'eliminating the contradiction': by *reduction to absurd*, or in other ways. I specified this point in my *Paradossi* (Roma: Carocci, 2009).

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(semantic, set-theoretic) paradoxes, even if these are considered from an epistemic point of view.

Let's see first the preface paradox. In its classical interpretation, it is a *weakened* Liar (according to the terminology proposed by van Fraassen, 1970): the author writes in the preface 'there is some false proposition in this book', and if there is something wrong in the book, there is no problem, if everything is right, the preface statement is false if true and true if false, so we have a standard Liar-like paradox. In another and more interesting interpretation (see Priest 2006) the case involves a true epistemic contradiction. The author believes *each* proposition of the book is true (otherwise he wouldn't have written it); and yet, as it seems, he does not believe they are *altogether* true. In other terms, suppose the book contains three sentences: the author believes that $T<p_1> \wedge T<p_2> \wedge T<p_3>$, but he also believes that $\neg T<p_1 \wedge p_2 \wedge p_3>$. Contradiction.

The probabilistic account tells us the paradox is solvable, or rather, there isn't any paradox at all (see Beall and Restall 2006). The author believes that $T<p_1> \wedge T<p_2> \wedge T<p_3>$ because, say, her evaluations are: $\nu(p_1) = 0.8$, $\nu(p_2) = 0.9$, $\nu(p_3) = 0.8$. But she does not believe that $T<p_1 \wedge p_2 \wedge p_3>$, because $\nu(p_1 \wedge p_2 \wedge p_3) = 0.8 \times 0.9 \times 0.8 = 0.576$, which is by far less compelling. More specifically, let's assume the author's epistemic standard is:

$$T<p> \text{ iff } P(p|e) \geq 0.8$$

(p is true if and only if its probability, on evidence given *e*, is 0.8 or more): we see our author is perfectly justified in accepting each p_n (0.8 or more) but not their conjunction (about 0.5).

The same account can be extended to other paradoxes involving inconsistency between distributive and conjunctive beliefs. See the *lottery paradox*: I do not really believe my ticket will win, as the winning ticket is 1/1,000; more properly, for any single t_n I do not believe that t_n will win, so I believe that $\neg W\, t_1 \wedge \neg W\, t_2 \wedge \neg W\, t_3 \ldots$; but I buy a ticket, because I believe that $W\, t_1 \vee W\, t_2 \vee W\, t_3 \ldots . \vee W_{1000}$. And I am perfectly entitled to rationally believe this.

This clearly validates the so-called *non adjunctive* approaches to contradictions, according to which (see Berto, 2007) we may have that p and $\neg$p are accepted, but this does not mean that $p \wedge \neg p$ is accepted as well. The Adjunction rule ($p, q \vdash p \wedge q$) fails. The failure can be explained in various ways, but, like Varzi (2004) concedes, these explanations are not unquestionable: if the adjunction rule does not hold, we cannot say we are properly speaking of '$\wedge$': one would say we are «changing the subject». Now in virtue of the probabilistic approach we can see well that in some cases to believe that p is true and q is true (0.8 or more) does not mean to rationally believe that $p \wedge q$ (is true). Our epistemic use of '$\wedge$' and similar operators is rational, though renouncing adjunction.

2. Truth value gluts are in fact truth value gaps

Beliefs are *contrastive*, in principle. If I believe that p (is true), usually this is because I do not believe that say q, or r, because they imply non-p.[2] More specifically: if I believe that p is true, I believe that non-p is false. Accordingly, in probabilistic logics for any α, if $P(\alpha) = 0.8$, then $P(\neg\alpha) = 0.2$: the classical meaning of negation as complementation is saved. More significantly, maintaining the epistemic standard at the specified level, we can have probabilistic truth value gaps without getting rid of the Excluded Middle. If for instance my valuation of p is 0.6, then $\nu(\neg p)$ will be 0.4, which means p and $\neg p$ are (to me) both untrue; but EM is preserved, as $\nu(p \vee \neg\vee) = P(p) + P(\neg p) = 0.6 + 0.4 = 1$.

Now paradoxical cases are alleged to be those, in which –supposedly– contrastivity does fail, and we have $\nu(p) \geq 0.8$ and $\nu(\neg p) \geq 0.8$ as well, so that $\nu(p \vee \neg p) > 1$. When and how can this happen? I suppose: *never*, but let's consider two controversial cases.

The reliabilist paradox - A strong reliabilist would claim that epistemic constraints depend on reliability. So it may happen that from a totally reliable source (say: my sense data) I get that p, and from another totally reliable source (say: logic) I get that not p. My valuations will be: $\nu(p) = 0.8$ or more, and $\nu(\neg p) = 0.8$ or more. Let's see α = 'α has the same truth value of β' and β = 'horses can fly'. If α is true, then β is true, and if α is false, then β is true; this is logic: but we know horses cannot fly (or so it seems).

Conflicts of reliable sources are usual, in political life. See a recent Italian case:

The Prime Minister says that p (p = 'Ruby is Mubarak's nephew')
It is unbelievable that a Prime Minister officially says something false
Therefore: p.

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[2] I may be wrong, for instance I may believe that p and that r, *without knowing* that $r \rightarrow \neg p$. But evidently, this does not regard *my* beliefs: I do not *believe* that $r \rightarrow \neg p$. For the connections between graded (probabilistic) and binary models of beliefs see Christensen 2007: 12-32.

Documents say that not p
It is unbelievable that official documents lie
Therefore: not p.

Possibly, there is no true conflict here (it is fairly evident where the truth is).[3] But there are typical reliability conflicts, in democratic contexts. See a canonical example. A Catholic rational believer will be perfectly confident in the Pope's infallibility, as well as in scientists' reliability. In virtue of the Pope's judgement, the proposition p = 'a 14 day human proto-embryo is a human being' is to be valued 0.8 or more (as catholic metaphysics does not admit of any substantial change in a human organism life); in virtue of the scientists' judgement, $\nu(\neg p) \geq 0.8$, so taking stem cells from a 14 day human proto-embryo is legitimate: it is not the same as killing a human being.[4] The two sources conflict.

Now the point is: what is it for contrastivity, in these cases? In fact, there is no reason to do away with it. As a matter of fact, if I have $\nu(p) = 0.8$ and $\nu(\neg p) = 0.8$, it is because I also have $\nu(p) = 0.2$ and $\nu(\neg p) = 0.2$. So I have *four* possible valuations. This means, in the dynamic of beliefs, that the Pope's reliability will *weaken* scientists' reliability, and the latter will operate in the opposite way. So we do not have properly a paradox, or an epistemic dilemma, but rather a general weakening of belief (and reliability).

We can see that in these cases over-determinacy (truth value glut) is transitory, and apparent, and easily turns into under-determinacy (truth value gap). Very simply, on that specific topic the rational Catholic believer will conclude she hasn't any clear idea of the matter, and p and not p are both (for her) untrue.[5]

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[3] It is also evident that Berlusconi, the Prime Minister of the case given, was not really persuaded that p. And yet, 341 members of the Italian Parliament voted in favour of the official version of the story: that he did not know that Ruby was not Mubarak's nephew – and he didn't know she was underage either.

[4] I consider here *human being* and *person* as equivalent terms. Notably, *non-Adj* strategy cannot be used in this case. The two sources correspond to different systems, so *true-for-the Pope* is to be distinguished from *true for biologists* (truth relativism); or also: *human being* in the Pope's language should be distinguished from biologists' *human being* (conceptual relativism). But in the believer's perspective, this is not a solution: because the two systems jointly act, in his own stock of beliefs.

[5] In fact, the classical inference from '$Tp \wedge Fp$' to '$\neg Tp \wedge \neg Fp$' is not perfectly justified, as it requires a doubtful use of the T-schema, as Priest 2007 very clearly shows: but I am suggesting here it is perfectly justified, instead, on *epistemic* level.

Fermi-Hart paradox - A similar situation occurs when conflicting pieces of evidence are involved (so the contradiction seems to regard things as they stand). Let's consider *Fermi-Hart argument* about extra-terrestrial intelligent life, also called of the «great silence»:

(1)
If advanced extraterrestrial civilizations existed, they would have contacted us
There is no rational and clear evidence that such a contact has taken place
Therefore: it is highly improbable that civilizations of this kind do exist.

The argument has been widely and variously discussed in the literature. But the main point is that we also have the other reasonable argument:

(2)
The universe is enormous or even infinite
Therefore: it is highly probable that advanced extraterrestrial civilizations do exist.

Again, we have four evaluations, actually. The statement p = 'there is some extraterrestrial intelligent life', seems to be 0.8 true, in virtue of (2), and 0.2true, in virtue of (1); and the same will be for $\neg$p. A possible dialetheist account on this subject would propose that p and not p are to be valued both true and false, and so their conjunction. However, on closer inspection, '$p \wedge \neg p$' is not *believed* as being true-and-false. Nobody would accept that extra-terrestrial intelligent life there is and there is not, «at the same time and under the same respect»; so whatever could be the epistemic agent's opinion on the matter, she will surely believe that $p \wedge \neg p$ is only false.

We might be tempted to apply a non-adjunctive approach. But consider the difference from the *Preface paradox*: in that case the author strictly admits that p_1 *is* (probably) *true*, and so are p_2, and p_3, etc.; in this case instead the epistemic agent would possibly take into account both arguments, but he won't properly *believe* that p, and he *won't believe* that not p either. Because he *knows* there might be two conflicting valuations of p, but these won't be *her* valuations. Rather, we may suppose that the consideration of each argument will give her reasons to diminishing the acceptability of the other.

3. Liars and probable truth

The most well known (allegedly) *true* contradictions are those conveyed by antinomies, to say: semantic and set-theoretic paradoxes, such as 'heterological' or Russell's class. The sentence μ says: ''μ' is false'. If 'μ' is true then it is false, and if it is false, it is true. So: $\mu \leftrightarrow \neg\mu$. The schema $\mu \leftrightarrow \neg\mu$ consists of $\mu \rightarrow \neg\mu$, and $\neg\mu \rightarrow \mu$, and classically (see Sainsbury 1995), for any α, if α implies the negation of α, non-α necessarily obtains; and if the negation of α implies α, then α is necessarily (analytically) true.[6] Now if α is *analytical* (or self-grounded) the valuation of α will be surely = 1, and if α is *self-refuting*, $\nu(\neg\alpha) = 1$. Thence, as it seems, $\nu(\alpha) + \nu(\neg\alpha) = 2$ (?!).

However, this cannot properly be an *epistemic* valuation. I cannot say that in virtue of $\neg\mu \rightarrow \mu$, I will strongly *believe* that μ (is true); and I cannot say that in virtue of $\mu \rightarrow \neg\mu$ I will strongly *believe* that ¬μ. In front of a Liar-like contradiction, I do not strictly believe each term of the contradiction is true.

Note that this is not due to *contrastivity*, like in case of the reliabilist paradox, and Fermi-Hart paradox. In those cases, the apparent soundness of opposite arguments yielded a positive diminishing of belief. In the Liar case, we ought to say that μ is true in virtue of ¬μ, and ¬μ is true in virtue of μ: each term confirms and disproves the other, at the same time. Not only that, truth value gap probabilistic strategies are perfectly justified when we have lack of information, but we cannot say Liar-like cases are of this sort. Actually, the information we have about the Liar's sentence, or 'heterological', or Russell's class, is perfectly *complete*. We do not need to know anything else (as it happens for extra-terrestrial intelligent life, or human 14 day proto-embryos). Logic gives us the adequate (categorical) grade of truth.

Another possible account might be that μ (insofar as mistaken or non well formed) is not something that can be epistemically true or false, and can consequently *be believed* true or false. And yet, this solution does not work, not only because of the famous Liar's revenge and other problems, but also because ultimately *we positively have a certain amount of beliefs concerning the Liar's and similar sentences*, and the point is precisely to specify what our epistemic attitude towards them is.

A more interesting analysis, I think, would suggest that *I do not believe that μ or ¬μ either*, because I positively know that μ implies ¬μ and vice versa; *but I believe that* $\mu \wedge \neg\mu$, because I see there are *logical facts*, which *make*

[6] See also Field's *Basic argument from equivalence to contradiction* (Field, 2008): given $\mu \leftrightarrow \neg\mu$, if μ then ¬μ, and therefore: $\mu \wedge \neg\mu$; correlatively, if ¬μ then μ, and therefore: $\mu \wedge \neg\mu$. So if the EM holds: $\mu \leftrightarrow \neg\mu \vdash \mu \wedge \neg\mu$.

true '$\mu \wedge \neg\mu$'. So I am in a situation, which is the opposite of the preface paradox: I believe that $\mu \wedge \neg\mu$, without believing μ, or $\neg\mu$ either. So $B(\mu \wedge \neg\mu)$, but $\neg B\mu$ and $\neg B\neg\mu$. *The contradiction is accepted, but only in its conjunctive form*.

The underlying question is: what is the meaning of 'truth' involved in these cases? It is quite evident that in the first three cases we deal with *epistemic* T, which (at least in these sorts of controversial contexts) is typically *incomplete*, and so perfectly adaptable to probabilistic logics. In the last case, we deal instead with *logical* T, because there are logical facts whose evidence makes me believe that $\mu \wedge \neg\mu$. Logical T is *absolute*, like *metaphysical* T (which possibly corresponds to the true meaning of 'T'). But are logical facts truly *facts*? This is the point, evidently.[7]

REFERENCES

BEALL, J.C. (2009): *Spandrels of Truth*, Oxford: Clarendon

BEALL, J.C. and RESTALL, G. (2006): *Logical Pluralism*, Oxford: Clarendon

BERTO, F. (2007): *How To Sell A Contradiction*, London: King's College

CHRISTENSEN, D. (2007): *Putting Logics in Its Place*, Clarendon: Oxford

FIELD, H. (2008): *Saving Truth From Paradox*, Oxford: Oxford University Press

PRIEST, G. (2006): *In Contradiction*, Oxford: Clarendon (2nd edition)

PRIEST, G. (2007): *Revenge, Field, and ZF*, In JC Beall, ed., *Revenge of the Liar*, Oxford: Oxford University Press

SAINSBURY, M. (1995): *Paradoxes*, Cambridge: Cambridge University Press

VAN FRAASSEN, B. (1970): *Truth and Paradoxical consequence*, in R. Martin, ed., *The Paradox of the Liar*, New Haven: Yale University Press

VARZI, A. (2004): *Conjunction and Contradiction*, in G. Priest et al., eds., *The Law of Non-Contradiction*, Oxford: Clarendon

[7] On the whole, I tend to favour *semantic* dialetheism: see Beall, 2009.