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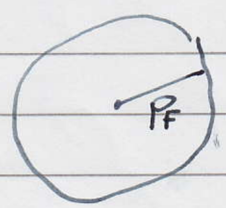
① $$\psi_{\bar p,\sigma} = \frac{e^{i\bar p\cdot\bar r/\hbar}}{\sqrt{A}}\underbrace{\chi_\sigma}_{\text{autofunzione dello spin } (\sigma=\pm1)} \qquad A = L^2$$

P.B.C.: $\bar p = \frac{2\pi\hbar}{L}(n_1, n_2)$ $\quad n_1, n_2 \in Z$

$$\epsilon_{\bar p,\sigma} = \frac{p^2}{2m}$$

② Tutti gli stati con fissato modulo di $\bar p$, $p = |\bar p|$, hanno uguale energia: occupiamo tutti gli stati all'interno del cerchio di raggio p_F, che contiene $N/2$ valori di $\bar p$; ciascun $\bar p$ "occupa" un'area $(2\pi\hbar/L)^2 = (h/L)^2$ nello spazio degli impulsi (P.B.C.)

$$N = \underbrace{2}_{\text{degenerazione di spin } (\sigma=\pm1)} \cdot \frac{\pi p_F^2}{(h/L)^2}$$



$$p_F^2 = \frac{N}{A}\frac{h^2}{2\pi} = n\frac{h^2}{2\pi} = 2\pi n\hbar^2$$

$$\boxed{p_F = \sqrt{2\pi n}\,\hbar} = \sqrt{\frac{n}{2\pi}}\,h$$

③ $$E = \sum_{\substack{\sigma=\pm 1 \\ |\bar{p}|<p_F}} \frac{p^2}{2m} = 2 \sum_{|\bar{p}|<p_F} \frac{p^2}{2m}$$

$$= \cancel{2} \int_{p<p_F} \frac{d\bar{p}}{h^2/A} \frac{p^2}{\cancel{2}m} = \frac{2\pi A}{h^2 m} \int_0^{p_F} dp\, p\, p^2$$

$$= \frac{2\pi A}{h^2 m} \frac{p_F^4}{4} = \frac{2\pi A}{h^2 m} \frac{p_F^2}{4} p_F^2$$

$$= \frac{\cancel{2\pi A}}{\cancel{h^2} m} \frac{N}{\cancel{A}} \frac{\cancel{h^2}}{\cancel{2\pi}} \frac{1}{4} p_F^2 = \frac{p_F^2}{4m} N = N \cdot \frac{h^2}{8\pi m} \frac{N}{A}$$

$$\boxed{E = \frac{1}{2} \epsilon_F \cdot N}$$

Nota $\epsilon < \epsilon_F \Rightarrow p < p_F$

④ $$\mu = \frac{\partial E}{\partial N} \qquad E \sim N^2$$

$$\boxed{\mu = \frac{2E}{N} = \epsilon_F} = \frac{p_F^2}{2m} = \frac{2\pi n \hbar^2}{2m} = \frac{\pi n \hbar^2}{m}$$

$$\mu = \frac{3.14 \times 10^{10} \times (1.05 \cdot 10^{-27})^2}{9.11 \cdot 10^{-28} \times 0.067} = 5{,}67 \times 10^{-16}\ \text{erg}$$

$$\boxed{\mu = 5.67 \times 10^{-16}\ \text{erg} = 3.54 \times 10^{-4}\ \text{eV} = 4.12\ ^\circ\text{K}}$$

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① Le energie sono date da

$$E_{\{n_{\vec{k}s}\}} = \sum_{s\vec{k}} \hbar\omega_{\vec{k}s} n_{\vec{k}s}$$

e sono fissate dagli interi $\{n_{\vec{k}s}\}$

$$Q = \sum_{\{n_{\vec{k}s}\}} e^{-\beta E_{\{n_{\vec{k}s}\}}} = \sum_{\{n_{\vec{k}s}\}} e^{-\sum_{s\vec{k}} \beta\hbar\omega_{\vec{k}s} n_{\vec{k}s}}$$

$$= \sum_{\{n_{\vec{k}s}\}} \prod_{s\vec{k}} e^{-\beta\hbar\omega_{s\vec{k}} n_{\vec{k}s}}$$

$$= \prod_{s\vec{k}} \sum_{n_{\vec{k}s}=0}^{\infty} e^{-\beta\hbar\omega_{\vec{k}s} n_{\vec{k}s}}$$

$$Q = \prod_{s\vec{k}} \frac{1}{1-e^{-\beta\hbar\omega_{\vec{k}s}}}$$

② $$E = \frac{\partial \beta A}{\partial \beta} = -\frac{\partial \ln Q}{\partial \beta} = +\frac{\partial}{\partial \beta} \sum_{s\vec{k}} \ln\left[1-e^{-\beta\hbar\omega_{\vec{k}s}}\right]$$

$$E = \sum_{s\vec{k}} \frac{e^{-\beta\hbar\omega_{\vec{k}s}}}{1-e^{-\beta\hbar\omega_{\vec{k}s}}} \cdot \hbar\omega_{\vec{k}s}$$

$$\boxed{E = \sum_{s\vec{k}} \frac{\hbar c k}{e^{\beta\hbar c k}-1}}$$

$$E = 2 \int_0^{k_D} \frac{d\bar{k}}{(2\pi)^2/A} \frac{\hbar c k}{e^{\beta\hbar c k} - 1}$$

$$= 2 \cdot \frac{2\pi}{(2\pi)^2/A} \int_0^{k_D} dk\, k \frac{\hbar c k}{e^{\beta\hbar c k} - 1} \qquad t = \beta\hbar c k$$

$$E = \frac{A}{\pi} \frac{(k_B T)^3}{(\hbar c)^2} \int_0^{\beta\hbar c k_D} dt \frac{t^2}{e^t - 1} \qquad \beta\hbar c k_D = \frac{T_D}{T}$$

$$\boxed{E = \frac{A}{\pi} \frac{(k_B T)^3}{(\hbar c)^2} \int_0^{T_D/T} dt \frac{t^2}{e^t - 1}}$$

$$T \to 0 \qquad \boxed{E \simeq \frac{A}{\pi} \frac{(k_B T)^3}{(\hbar c)^2} \cdot \zeta(3)}$$

(3) $$c_V = \frac{\partial E/A}{\partial T} = \frac{3}{\pi} \frac{k_B^3 T^2}{(\hbar c)^2} \zeta(3)$$

(4) $$N = \frac{\pi k_D^2}{(2\pi)^2/A} \qquad \boxed{k_D^2 = 4\pi \frac{N}{A} = 4\pi n}$$

$$k_D = 2\sqrt{\pi n} = 2\sqrt{3.14 \times 10^9}$$

$$\boxed{k_D = 1.12 \times 10^5 \ cm^{-1}}$$