ESERCIZIO ~~2~~ 1 ~~17/02/2011~~ 21/12/12

①

PBC: $\bar{p} = \frac{2\pi}{L}\hbar(n_1, n_2) \qquad n_1, n_2 \in Z$

Poiché l'energia dipende dal solo modulo $p = |\bar{p}|$ e cresce con p, occuperemo gli stati per cerchi di raggio crescente fino al cerchio massimo

$$N = \underbrace{2}_{\text{deg. di spin}} \frac{\pi p_F^2}{(h/L)^2}$$

$$p_F^2 = \frac{N}{A}\frac{h^2}{2\pi} = 2\pi n \hbar^2 \qquad n = \frac{N}{A} = \frac{N}{L^2}$$

$$\boxed{p_F = \hbar\sqrt{2\pi n}}$$

② $E = 2\sum_{\bar{p}} \alpha p = 2\frac{1}{(h/L)^2}\int_{p<p_F} d\bar{p}\, \alpha p$

$= \frac{2A}{h^2} 2\pi\alpha \int_0^{p_F} dp\, p\, p = \frac{4\pi\alpha}{h^2} A \frac{p_F^3}{3}$

$= \frac{4\pi}{h^2} A \frac{p_F^2}{3}\alpha p_F = \frac{2\cancel{4}\cancel{\pi}}{\cancel{h^2}} \cancel{A} \frac{N}{\cancel{A}}\frac{\cancel{h^2}}{\cancel{2\pi}}\frac{1}{3}\alpha p_F$

$$\boxed{E = N\frac{2}{3}\alpha p_F = N\frac{2}{3}\epsilon_F}$$

(3)

$$\mu = \frac{\partial E}{\partial N} = \frac{\partial}{\partial N}\left[\frac{2}{3}\alpha N\sqrt{\frac{h^2}{2\pi}\frac{N}{A}}\right]$$

$$= \frac{3}{2}\cdot\frac{2}{3}\alpha\frac{N}{N}\sqrt{\frac{h}{2\pi}\frac{N}{A}} = \alpha p_F$$

$$\boxed{\mu = \alpha p_F = \epsilon_F}$$

$\boxed{\text{c.g.s}}$

$$\mu = 10^8 \cdot 1{,}05\times10^{-27}\sqrt{2\pi\cdot10^{12}}$$

$$= 1.05\times10^{-13}\sqrt{6.28} = 2.63\times10^{-13}\,\text{erg}$$

$$= 0{,}164\,\text{eV}$$

$$\boxed{\mu = 0{,}164\,\text{eV} = 2.63\times10^{-13}\,\text{erg}}$$

$$\boxed{\mu = 2.63\times10^{-20}\,\text{J}}$$

(4)

$$P = -\frac{\partial E}{\partial A} = -\frac{\partial}{\partial A}\left[N\frac{2}{3}\alpha\sqrt{\frac{h^2}{2\pi}\frac{N}{A}}\right]$$

$$= \frac{1}{2}\frac{N}{A}\frac{2}{3}\alpha\sqrt{\frac{h^2N}{2\pi A}} = \frac{1}{3}n\alpha p_F$$

$$\boxed{P = \frac{1}{2}\frac{E}{A} = \frac{1}{3}n\epsilon_F}$$

Esercizio 2 21/12/2012

(1) $$\rho = \frac{\langle N \rangle}{L} = \frac{1}{L} \sum_p \frac{1}{\frac{e^{\beta p^2/2m}}{z} - 1}$$

$$= \frac{1}{\cancel{L}} \int_{-\infty}^{+\infty} \frac{dp}{\frac{h}{\cancel{L}}} \frac{z e^{-\beta p^2/2m}}{1 - z e^{-\beta p^2/2m}} \qquad t = \sqrt{\frac{\beta}{2m}} \cdot p$$

$$= \frac{\sqrt{2mk_BT}}{h} \int_{-\infty}^{+\infty} dt \, \frac{z e^{-t^2}}{1 - z e^{-t^2}}$$

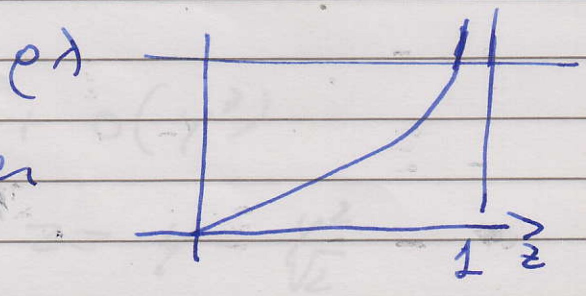
$$= \frac{1}{\lambda\sqrt{\pi}} \, 2 \int_0^{\infty} dt \, z e^{-t^2} \sum_{n=0}^{\infty} \left(z e^{-t^2}\right)^n$$

$$= \frac{2}{\sqrt{\pi}\,\lambda} \sum_{n=1}^{\infty} z^n \int_0^{\infty} dt \, e^{-nt^2}$$

$$= \frac{2}{\sqrt{\pi}\,\lambda} \sum_{n=1}^{\infty} z^n \, \frac{1}{2} \sqrt{\frac{\pi}{n}} = \frac{1}{\lambda} \sum_{n=1}^{\infty} \frac{z^n}{n^{\frac{1}{2}}}$$

$$\boxed{\rho = \frac{1}{\lambda} \, g_{1/2}(z)}$$

(2) $$g\left(\tfrac{1}{2}\right) = \sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{2}}} = \infty$$



- c'è sempre soluzione per $z(\rho\lambda)$ per qualsiasi $\rho\lambda$ finito con $z<1$
- non c'è condensazione!

③

$$\beta P = \frac{1}{L} \ln Z = \frac{1}{L}\left\{ -\sum_p \ln\left(1 - z e^{-\beta p^2/2m}\right)\right\}$$

$$= -\frac{1}{\Delta}\int_{-\infty}^{+\infty} \frac{dp}{\frac{h}{L}} \ln\left(1 - z e^{-\beta p^2/2m}\right)$$

$$= -\frac{\sqrt{2mk_BT}}{h}\int_{-\infty}^{\infty} dt \ln\left(1 - z e^{-t^2}\right)$$

$$= +\frac{2}{\sqrt{\pi}\,\lambda}\int_0^{\infty} dt \sum_{n=1}^{\infty} \frac{(z e^{-t^2})^n}{n}$$

$$= +\frac{2}{\sqrt{\pi}\,\lambda}\sum_{n=1}^{\infty}\frac{z^n}{n}\int_0^{\infty} dt\, e^{-nt^2}$$

$$= +\frac{2}{\sqrt{\pi}\,\lambda}\sum_{n=1}^{\infty}\frac{z^n}{n}\,\frac{1}{2}\sqrt{\frac{\pi}{n}} = \frac{1}{\lambda}\sum_{n=1}^{\infty}\frac{z^n}{n^{3/2}}$$

$$\boxed{\beta P = \frac{1}{\lambda} g_{3/2}(z)}$$

④ $y = \lambda \rho = \quad z + \frac{z^2}{\sqrt{2}} + \dots$

$z = y + a y^2 + \dots$

$y = y + a y^2 + \frac{y^2}{\sqrt{2}} + o(y^3)$

$\Rightarrow \quad a = -\frac{1}{\sqrt{2}} \qquad z = y - \frac{y^2}{\sqrt{2}}$

$$\beta P = \frac{1}{\lambda}\left[z + \frac{z^2}{2^{3/2}} + \cdots \right]$$

$$\simeq \frac{1}{\lambda}\left[\lambda\rho - \frac{(\lambda\rho)^2}{\sqrt{2}} + \frac{(\lambda\rho)^2}{2^{3/2}} + O(\lambda\rho)^3 \right]$$

$$= \frac{1}{\lambda}\left[\lambda\rho - \frac{(\lambda\rho)^2}{2^{3/2}}\left[2-1\right] + O(\lambda\rho)^3 \right]$$

$$= \frac{1}{\lambda}\left[\lambda\rho - \frac{(\lambda\rho)^2}{2^{3/2}} \right]$$

$$\boxed{\beta P = \rho\left[1 - \frac{\lambda\rho}{2^{3/2}} + O(\lambda\rho)^2 \right]}$$