Condensed Matter Physics II. – A.A. 2015-2016, April 29, 2016

(*time 3 hours*)

Solve the following two exercises.

NOTE:

- Give all details which help in understanding the proposed solution. Answers which only contain the final result or not enough detail will be judged insufficient and discarded;
- If you are requested to give evaluation/estimates, do so using 3 significant figures.

Exercise 1: *Stoner model in 1 D*

Consider Fermions of spin 1/2 in 1D with a pair interaction that in atomic units ($\hbar = m = e = 1$) reads $(U/n)\delta(x_1 - x_2)$, with the constant U the strengh of the repulsive interaction and n the linear density. A magnetic field along x is also present, so that the Hamiltonian is

$$\sum_i^N \left[-\frac{\nabla_i^2}{2} + 2\mu_B B S_{x,i} \right] + \frac{1}{2} \sum_{i \neq j} (U/n)\delta(x_i - x_j)$$

and $B > 0$. Consider the HF approximation with a Slater determinant with $N_\uparrow$ Fermions with spin up, which occupy the plane waves of the Fermi segment $|k| \leq K_{F,\uparrow}$, and $N_\downarrow$ Fermions with spin down, which occupy the plane waves of the Fermi segment $|k| \leq K_{F,\downarrow}$.

1. Calculate the kinetic energy per Fermion.
2. Try to calculate the interaction energy among the Fermions; you should get an energy per Fermion
$$U \frac{N_\uparrow N_\downarrow}{N^2}.$$
3. Combining the energy of the previous two points with the interaction energy of spins with the magnetic field get the total energy per Fermion $e(n, \zeta, B)$, as function of the areal density $n = n_\uparrow + n_\downarrow$, the spin polarization $\zeta = (n_\uparrow - n_\downarrow)/n$, and the field B.
4. Get the equilibrium $\zeta(n, B)$, i.e., the polarization that minimizes the total energy at given n, B. Remember that $-1 \leq \zeta \leq 1$ and that to find the absolute minimum of a function on a segment one looks at first and second derivatives of the function as well as to the values of the function on the boundary.
5. What's the expression of $\zeta(n, B)$ at large density?
6. Wht's the value of $\zeta(n, B)$ for small density?

Esercizio 2: Carrier densities in a two-dimensional semiconductor

Consider a 2 dimensional crystal, which at $T = 0$ has some completely full energy bands, the others being empty. The energy gap between the uppermost full energy band and the first empty one is $E_g = 2.15eV$. Let's treat this system as an intrinsic 2 dimensional semiconductor, neglecting the effect of impurities.

The energy dispersion at the top of the valence band is è

$$\epsilon_v(\mathbf{k}) = \epsilon_v - \frac{\hbar^2}{2m_v}\mathbf{k}^2 + ...$$

and at the bottom of the conduction band

$$\epsilon_c(\mathbf{k}) = \epsilon_c + \frac{\hbar^2}{2}\left(\frac{k_x^2}{m_x} + \frac{k_y^2}{m_y}\right) + ...$$

1. Calculate the energy density of states at the top of the valence band.
2. Calculate the energy density of states at the bottom of the conduction band.
3. Assume the non-degenerate regime and temperature $K_B T$ much smaller of the bands width, in order to be able to use eqs. (28.12-13) of the textbook and the energy density of states obtained earlier. Calculate $N_c(T)$.
4. Calculate $P_v(T)$.
5. Knowing that $m_v = 0.4m_e$, $m_x = 0.2m_e$, and $m_y = m_e$ evaluate numerically $N_c(T)$ e $P_v(T)$ at room temperature ($T = 300\,^oK$).
6. Evaluate numerically the intrinsic carrier density $n_i(T)$ at room temperature.