Condensed Matter Physics II. – A.A. 2009-20010, June 7 2010

Second partial exam

(*time 3 hours*)

Solve the following two exercises, each has a maximum score of 18 for a total of 36. A score between 33 e 36 corresponds to 30 cum laude, between 30 e 32 is renormalized to 30 (the maximum official score, without laude).

NOTA BENE:

- Give all details which help in understanding the proposed solution. Answers which only contain the final result or not enough detail will be judged insufficient and discarded;
- If you are requested to give evaluation/estimates, do so using 3 significant figures.

Esercizio 1: *One-dimensional linear chain with interactions beyond the nearest neighbours.*

Considerino the oscillations of a linear atomic chain, with interatomic distance a and mass M, when *harmonic* interactions beyond the nearest neighbours are present.

1. Write the potential energy of the chain with $u(na)$ the displacement from equilibrium position na of the n-th atom and C_m the force constant of order m, $m \geq 1$. Recall that the interaction energy between the n-th atom and its m-th neighbour is $(1/2)C_m\left[u(n) - u(n+m)\right]^2$.
2. Obtain the force acting on the l-th atom.
3. Obtain the condition under which a solution of the type $e^{i(qal-\omega t)}$ satisfies the equations of motions, i.e., obtain the dispersion relation for the frequency, $\omega(q)$.
4. Expand the dispersion relation for small values of q to obtain an expression for the sound velocity. Which is the condition that the C_m must satisfy in order to obtain a finite sound velocity?
5. Consider now and only now $C_m = C/m^p$ with $1 < p < 3$: what is the value of the sound velocity obtained earlier with such a choice of C_m?
6. With $C_m = C/m^p$, rather than expanding in q obtain the behaviour of $\omega(q)$ for q small by approximating the m sum in the dispersion relation with an integral [the approximation becomes exact in the limit $q \to 0$].

Exercise 2 : *Pauli Paramagnetism in a 2D electron gas.*

Consider a non interacting electron gas in 2 dimension.

1. Calculate (or just write it down if you know it) the energy density of state $g(\epsilon)$ for spin unpolarized electrons and express it in terms of the areal density $n = N/S$ and the Fermi energy ϵ_F. What are the dimensions of $g(\epsilon)$. Sketch $g(\epsilon)$, being carefull to the allowed energy range.

2. A magnetic field $\mathbf{H} = H\hat{\mathbf{z}}$ is applied to the electrons, which move in the plane (y,z). Write the interaction energy with the field for an electron with spin projection S_z (in units of $\hbar$). Take the electron anomaly (*g-factor*) as $g_0 = 2$ and express the results in terms of the Bohr magneton μ_B. [Note, as the electrons move in 2 dimension and the field is in plane, one has to consider only the coupling of the spin with the field, i.e., there are no orbital terms.]

3. Calculate the density of states of spin up and spin down electrons, $g_+(\epsilon)$ and $g_-(\epsilon)$), and sketch them.[Be careful to specify the allowed energy ranges for $g_+(\epsilon)$ and $g_-(\epsilon)$)].

4. Derive for the present case and for arbitrary values of H and T, the expression of the magnetization density induced by the field in terms of the Fermi function $f(\epsilon)$ and the densities of states $g_+(\epsilon)$ and $g_-(\epsilon)$).[Recall that the magnetization density is readily obtained from the partial densities n_+, n_-.]

5. Neglecting the dependence of the chemical potential μ on temperature and magnetic field and assuming that $\mu_B H \ll \epsilon_F$ and $K_B T \ll \epsilon_F$, calculate the magnetization density to leading order in H.

6. Give the expression of the resulting magnetic susceptibility and comment its relation with the known 3-dimensional result.