Condensed Matter Physics II. – A.A. 2012-2013, May 8 2013

(*time 3 hours*)

Solve the following two exercises, each has a maximum score of 18 for a total of 36. A score between 33 e 36 corresponds to 30 cum laude, between 30 e 32 is renormalized to 30 (the maximum official score, without laude).

NOTE:

- Give all details which help in understanding the proposed solution. Answers which only contain the final result or not enough detail will be judged insufficient and discarded;
- If you are requested to give evaluation/estimates, do so using 3 significant figures.

Exercise 1: *Stoner model in 2 D*

Consider Fermions of spin 1/2 in 2D with a pair interaction that in atomic units ($\hbar = m = e = 1$) reads $(U/n)\delta(\mathbf{r}_1 - \mathbf{r}_2)$, with the constant U the interaction strengh and n the areal density. A magnetic field along z is also present, so that the Hamiltonian is

$$\sum_i^N \left[-\frac{\nabla_i^2}{2} + 2\mu_B B S_{z,i} \right] + \frac{1}{2} \sum_{i \neq j} (U/n)\delta(\mathbf{r}_i - \mathbf{r}_j)$$

and $B > 0$. Consider the HF approximation with a Slater determinant with $N_\uparrow$ Fermions with spin up, which occupy the plane waves of a Fermi disk of radius $K_{F,\uparrow}$, and $N_\downarrow$ Fermions with spin down, which occupy the plane waves of a Fermi disk of radius $K_{F,\downarrow}$.

1. Calculate the kinetic energy per Fermion.
2. Try to calculate interaction energy among the Fermions; you should get an energy per Fermion
$$U \frac{N_\uparrow N_\downarrow}{N^2}.$$
3. Combining the energy of the previous two points with the interaction energy of spins with the magnetic field get the total energy per Fermion $e(n, \zeta, B)$, as function of the areal density $n = n_\uparrow + n_\downarrow$, the spin polarization $\zeta = (n_\uparrow - n_\downarrow)/n$, and the field B.
4. Get the equilibrium $\zeta(n, B)$, i.e., the polarization that minimizes the total energy at given n, B. Remember that $-1 \leq \zeta \leq 1$ and that to find the absolute minimum of a function on a segment one looks at first and second derivatives of the function as well as to the values of the function on the boundary.
5. What's the expression of $\zeta(n, B)$ at large density?
6. Wht's the value of $\zeta(n, B)$ for small density?

Exercise 2: *2D semiconductor on a triangular lattice*

Let us consider a 2D system whereby divalent atoms occupy the sites of a triangular lattice. Each atom releases 2 electrons, forming a divalent positive ion. The 2 bands lowest in energy have the tight-binding form:

$$E_v(\mathbf{q}) = 2\gamma_v(-3 + \cos(q_x a) + 2\cos(q_x a/2)\cos(q_y a\sqrt{3}/2),$$

$$E_c(\mathbf{q}) = 2\Delta - 2\gamma_c(-3 + \cos(q_x a) + 2\cos(q_x a/2)\cos(q_y a\sqrt{3}/2).$$

1. At $T = 0$ the system is a metal or an insulator and why?
2. Expand $E_v(\mathbf{q})$ [$E_c(\mathbf{q})$] near its maximum ([minimum] in q-space, to second order.
3. Using the previous result, calculate the density of states $g_v(E)$ [$g_v(E)$] of $E_v(\mathbf{q})$ [$E_c(\mathbf{q})$] near its maximum [minimum].
4. Assuming that the semiconductor is in the non-degenerate regime calculate the concentrations of electrons $n_c(T)$ and holes $p_v(T)$, respectivley in the conduction and valence band, at low temperature.
5. Imposing the equality $n_c(T) = p_v(T)$ (clean semiconductor) obtain $\mu(T)$.
6. Determine the value of γ_c/γ_v that, for small temperature, yields $\mu(T) = \Delta + 2K_BT$.