Condensed Matter Physics II. – A.A. 2010-2011, May 6 2011

(*time 3 hours)*

Solve the following two exercises, each has a maximum score of 18 for a total of 36. A score between 33 e 36 corresponds to 30 cum laude, between 30 e 32 is renormalized to 30 (the maximum official score, without laude).

NOTE:

- Give all details which help in understanding the proposed solution. Answers which only contain the final result or not enough detail will be judged insufficient and discarded;
- If you are requested to give evaluation/estimates, do so using 3 significant figures.

Esercizio 1: *LDA for 2D Fermions with linear energy dispersion*

Consider N non-interacting spin 1/2 Fermions in 2D (on the surface A) with single-particle energy dispersion $\epsilon_{\mathbf{p}} = \alpha p, \quad \alpha > 0$, and Periodic Boundary Conditions (PBC). Restrict to the spin unpolarized state (equal populations for up and down spin projections).

1. Calculate the Fermi wavevector q_F.
2. Express the Fermi energy E_F in terms of the areal number density $n = N/A$.
3. Calculate the ground state energy (total energy) of such system, and from it the energy per Fermion $\varepsilon(n)$.
4. Consider now the same system of Fermions in an external potential $v(\mathbf{r})$ and write the totale energy (kinetic + interaction with the external potential) resorting to the Local Density Approximation (LDA) for the kinetic energy.
5. Obtain the equilibrium density (from the minimum energy principle).
6. Specialize now to the case in which the Lagrange multiplier μ, introduced in the minimization, is set to 0 and $v(\mathbf{r}) = -v_0[sin(qr)/qr]^{1/2}$ for $r \leq \pi/q$ and $v(\mathbf{r}) = 0$ when $r > \pi/q$: calculate the total number of Fermions that correspond to the density profile obtained for $v_0 = [M/2]^{1/2}\alpha\hbar q$.

Esercizio 2: Carrier densities in a two-dimensional semiconductor

Consider a 2 dimensional crystal at $T = 0$ with some full energy bands, the others being empty. The energy gap between the uppermost full energy band and the first empty one is $E_g = 1eV$. Let's treat this system as an intrinsic 2 dimensional semiconductor, neglecting the effect of impurities.

The energy dispersion at the top of the valence band is è

$$\epsilon_v(\mathbf{k}) = \epsilon_v - \frac{\hbar^2}{2m_v}(\mathbf{k} - \mathbf{k}*)^2 + ...$$

and at the bottom of the conduction band

$$\epsilon_c(\mathbf{k}) = \epsilon_c + \frac{\hbar^2}{2m_c}(\mathbf{k} - \mathbf{k}*)^2 + ...$$

1. Calculate the energy density of states at the top of the valence band.
2. Calculate the energy density of states at the bottom of the conduction band.
3. Assume the non-degenerate regime and temperature K_BT much smaller of the bands width, in order to be able to use eqs. (28.12-13) of the textbook and the energy density of states obtained earlier. Calculate $N_c(T)$.
4. Calculate $P_v(T)$.
5. Knowing that $m_v = 0.38m_e$ and $m_c = 0.067m_e$, evaluate numerically $N_c(T)$ e $P_v(T)$ at room temperature ($T = 300\,^oK$).
6. Evaluate numerically the intrinsic carrier density $n_i(T)$ at room temperature.