

Numerical modeling of porous materials' mechanical behavior with the cell method

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Abstract

Cell method is a numerical method that has recently been developed. It allows a direct discrete formulation of physical laws, and therefore it is particularly suitable when heterogeneities are present, as in porous materials. The proposed model consists of a matrix of constituent cells with randomly distributed voids. Results of simulations in both elastic and plastic field show a good agreement with experimental data for sintered alloys.

Keywords: Cell method; Plasticity; Sintered alloys; Mechanical behavior; Porous material simulation

1. Introduction

Cell method (CM) is a numerical method that has recently been developed [1], and constitutes an alternative to FEM. It does not resort to a differential formulation in order to write equilibrium equations. This makes CM particularly suitable to model porous material mechanical behavior. A detailed description of CM for plane elasticity and comparisons with FEM results may be found in Ref. [2].

An aspect of some relevance when discontinuities are encountered in a problem is that a differentiation is not possible and special elements must be developed in order to overcome this problem if FEM is used: as a general statement, the use of FEM implies the size of the mesh to be smaller than any characteristic length involved in the problem [3].

On the contrary, the CM uses the *dual cell* concept, where the dual cell constitutes an influence region for each node (Fig.1), obtained by the Voronoi tessellation associated with the triangle, primal, mesh. Balance equations are written for each influence region. The result is a linear system in the usual form

$$\{F\} = [K]\{u\} \quad (1)$$

which has now been derived in a discrete form. Thus, no restriction is imposed by differentiability and the constitutive matrix may vary freely from one cell to the neighbor.

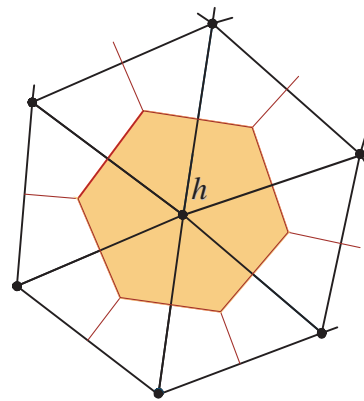


Fig. 1. Dual cell of node h .

This feature of CM has been here applied in order to model the mechanical behavior of sintered alloys.

2. Sintered alloys

Residual porosity strongly affects mechanical properties in sintered alloys, leading to a reduction of strength and ductility with respect to wrought materials [4]; as the presence of pores within the constituent reduces the effective section and induces stress concentrations.

An effort has been carried on in order to develop a numerical model, which may be used to predict mechanical properties of sintered alloys, given the residual porosity [5–9]. It employs CM and the preliminary results, discussed

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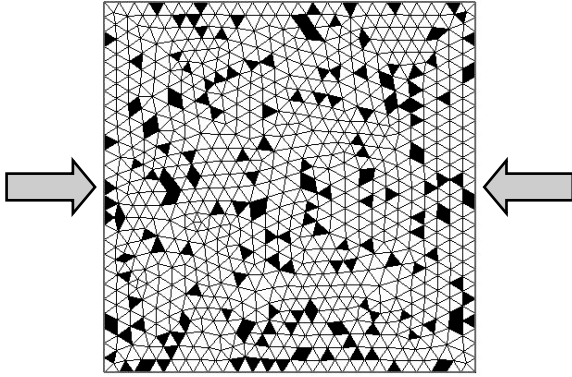


Fig. 2. A model with 1498 cells: void cells (black) among constituent (white).

in [5,6], although restricted to elastic field, were in good agreement with experimental data [10], deviation never been greater than 10%, which is within the usual range of variability for such materials.

The model consists of a matrix of cells (constituent cells), among which a number of void cells is randomly distributed, accounting for residual porosity, as shown in Fig. 2. CM is used to simulate compression tests on this model.

3. Simulations in plastic field

In order to widen the application field of the model, implementation of non-linear constitutive relationships is needed. The proposed model has therefore been extended in order to take into account for plasticity.

An elastic–perfectly plastic incremental model for CM has been developed and discussed in Refs. [11,12] and applied to the constituent cells in sintered alloys [8], indicating that hardening had to be included in the constituent behavior in plastic field.

Consequently, a new model has been developed [9]. For each constituent cell an elastic–plastic behavior with hardening has been assumed. Consistently with the approximation used in the elastic field, plastic strain components λ_c are uniform inside the primal cell. Load history is divided into a convenient number of finite steps. Displacements and internal stresses are known at the beginning of each step. The fundamental system (1) in incremental terms now reads

$$\{\Delta F\} = [K]\{\Delta u\} - [L]\{\Delta \lambda\} \quad (2)$$

where $[L]\{\Delta \lambda\}$ collects the plastic strain equivalent forces.

At step one the condition $\{\Delta \lambda\} = 0$ is assumed and system (2) is solved for Δu . For each primal cell, stress $\{\sigma\}_c$ and deviatoric strain $\{s\}_c$ can be then computed.

Von Mises yield condition is assumed and the radius of the yield locus is used to update $\Delta \lambda$:

$$\Delta r = \sqrt{s_x^2 + s_y^2 + s_z^2 + s_{xy}^2} - \sqrt{\frac{2}{3}} \sigma_Y \quad (3)$$

- $\Delta r < 0$: the cell is in elastic field, this is the solution;
- $\Delta r > 0$: Δr is used to update $\{\Delta \lambda\}_c$, $\{s\}_c$.

The process is repeated until convergence is obtained for all cells, then a new step is considered.

A ferrous powder has been modeled. Constituent is NC100.24, characterized by the following mechanical parameters:

- $E = 207$ GPa, elastic modulus in the first part of the stress/strain diagram,
- $H = 2$ GPa, tangential modulus in the second part of the stress/strain diagram,
- $\nu = 0.3$, Poisson's ratio of constituent in elastic field,
- $\nu = 0.5$, Poisson's ratio of constituent in plastic field,
- $R_y = 180$ MPa, yield stress.

A total number of 1498 cells were used for the model. The 13.4% residual porosity of the sintered alloy was modeled by 182 void cells, as shown in Fig. 1. Fig. 3 shows progressive plasticization during the simulation for this alloy.

The stress/strain plot obtained during the simulation is reported in Fig. 4. The effective Young modulus in the initial elastic field is 164 GPa, a result within the range of variability of the previous simulations and experimental data (experimental effective Young modulus was 150 GPa).

Beyond the elastic field, the stress/strain plot of experimental data for this alloy show in Ref. [10] a discontinuity in the region of yielding, and this behavior is reproduced in the simulation, where slope changes at yield.

Mechanical behavior of sintered alloys in the plastic field is often represented by Hollomon's equation, which is not suitable in the elastic region and is not able to account for the above-mentioned discontinuity. Hollomon's equation is also plotted in Fig. 4, using the parameters given in Ref. [10] that best fit the experimental data for the simulated alloy.

4. Conclusions

Although the number of simulations performed at the moment is small, these preliminary results show a very good agreement between simulations and experimental data.

The proposed numerical model is suitable for mechanical behavior of sintered alloys assessment depending on residual porosity. It considers both heterogeneities of the structure and stress concentrations.

The model consists of a matrix of constituent cells with randomly distributed voids and is solved with CM, which is particularly suitable for heterogeneous materials.

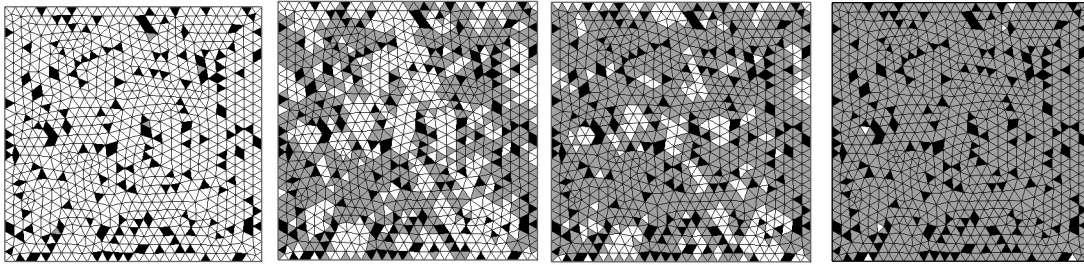


Fig. 3. Progressive plasticization of alloy A1: Void cells (black) among constituent cells in elastic (white) and in plastic (gray) field.

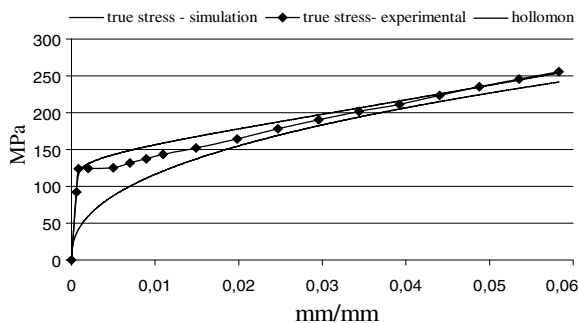


Fig. 4. Simulations results.

Mechanical behavior of the simulated sintered alloys could be assessed with very good accuracy, given the constituent mechanical parameters and the alloy residual porosity. Moreover, an advantage of these simulations against Hollomon's equations is that the latter cannot be used to predict the alloy behavior, as it requires experimental tests for parameters assessment and can therefore be used only a posteriori.

Application to a larger number of different sintered alloys is currently being carried on.

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