Near-Capacity STSK MIMO systems for Dispersive Channels

Tesi di Laurea in Trasmissione Numerica

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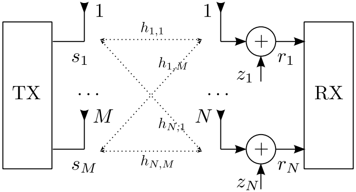
Objective of the thesis

- Analysing the most popular MIMO techniques such as STBCs and V-BLAST and the most recent MIMO schemes such as LDC, SM, SSK.
- Deeply studying the STSK scheme, in both coded and uncoded scenarios.
- Proposing new techniques and criteria for designing STSK schemes having good performance in both coded and uncoded scenarios.
- Designing an OFDM-aided STSK scheme for dispersive channels.

Outline

What MIMO techniques are



- MIMO is a promising technique for obtaining high data rates and reliability in wireless mobile channels.
- MIMO schemes employ:
  - multiple transmit and receive antennas;
  - appropriate space-time coding algorithms.
- MIMO can provide:
  - diversity: Alamouti scheme, STBCs;
  - multiplexing: V-BLAST;
  - a mixture of both: recent schemes such as STSK.

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MIMO channel model

- A $M \times N$ MIMO Rayleigh flat fading channel model:

$$\boldsymbol{H} \in \mathbb{C}^{N \times M} \quad \text{where} \quad h_{j,i} \sim \mathcal{CN}(0,1).$$

- Block fading channel having duration T may be modelled as:

$$\boldsymbol{R} = \boldsymbol{H}\boldsymbol{S} + \boldsymbol{Z},$$

  where $\boldsymbol{R}, \boldsymbol{Z} \in \mathbb{C}^{N \times T}$ and $\boldsymbol{S} \in \mathbb{C}^{M \times T}$.

- Energy constraints: $\mathbb{E}[\mathrm{tr}(\boldsymbol{S}\boldsymbol{S}^{\mathrm{H}})] = T$ and $z_{j,t} \sim \mathcal{CN}(0, \gamma_{\mathrm{AV}}^{-1})$.
- Telatar and Foschini demonstrated a significant capacity increase for the $M \times N$ MIMO channel.

$$C_{M \times N} \simeq \min\{M, N\} \log_2 \frac{\gamma_{\mathsf{AV}}}{M} \quad \text{if} \quad \gamma_{\mathsf{AV}} \to \infty$$

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Space-Time Block Codes

- STBCs are based on the concept of orthogonal designs.
- An orthogonal design is a complex matrix having as entries the unknown $\pm x_1, \cdots, \pm x_K$ and their conjugates and satisfying particular orthogonality properties.
- Examples of orthogonal designs are:

$$\mathcal{O}_{c,2} = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \in \mathbb{C}^{2\times 2}, \quad \text{(the well known Alamouti code)}$$

$$\mathcal{X}'_{\mathrm{c},3} = \begin{bmatrix} x_1 & x_2^* & x_3^* & 0 \\ -x_2 & x_1^* & 0 & -x_3^* \\ -x_3 & 0 & x_1^* & x_2^* \end{bmatrix} \in \mathbb{C}^{3\times 4},$$

$$\mathcal{G}_{\mathrm{c},3} = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & x_1^* & -x_2^* & -x_3^* & -x_4^* \\ x_2 & x_1 & x_4 & -x_3 & x_2^* & x_1^* & x_4^* & -x_3^* \\ x_3 & -x_4 & x_1 & x_2 & x_3^* & -x_4^* & x_1^* & x_2^* \end{bmatrix} \in \mathbb{C}^{3\times 8}.$$

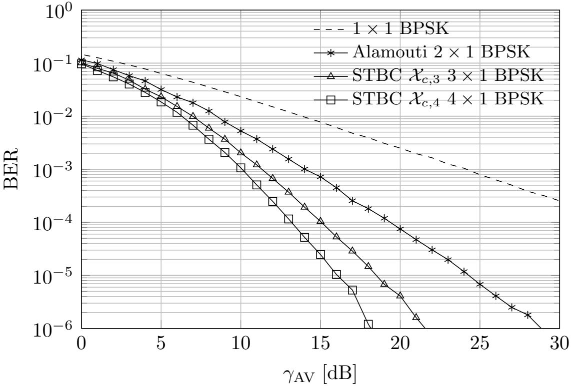
- The orthogonality property permits simple symbol-by-symbol decoding.

STBC performance and diversity gain

- The diversity gain ζ_{D} quantifies the BER performance gain:

$$\zeta_{\mathrm{D}} \equiv - \lim_{\gamma_{\mathrm{AV}} \to +\infty} \frac{\log \bar{P}_{\mathrm{e}}\left(\gamma_{\mathrm{AV}}\right)}{\log \gamma_{\mathrm{AV}}} \in \mathbb{N}^{+}.$$

- STBCs provide full diversity gain, i.e. $\zeta_{\mathrm{D}} = NM$, but have low coding rates.



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- STBCs and diversity gain
- V-BLAST and multiplexing gain
- Diversity-multiplexing trade-off
- Space-Time Shift Keying

Improving STSK performance
- Previously proposed design criteria
- A new design criterion based on EXIT charts
- Genetic algorithm aided design

STSK in wideband channels
- Wideband channels
- OFDM-aided STSK

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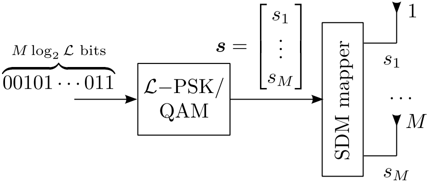
Vertical Bell Labs Layered Space-Time architecture



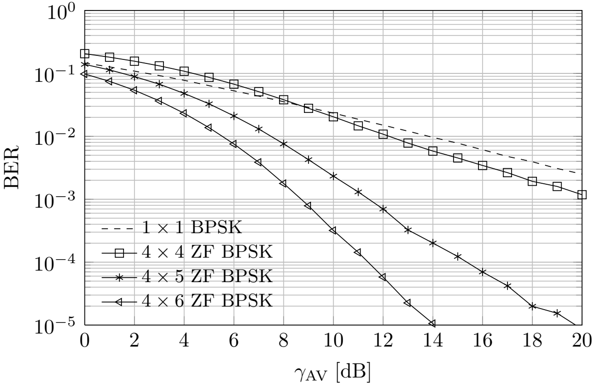
- V-BLAST: spatial division multiplexing system with simple decoder based on linear signal processing (only if $N \geq M$).
- M independent streams are transmitted simultaneously.
- Sub-optimal symbol-by-symbol decoding based on:
  - interference nulling (ZF or MMSE);
  - symbol cancellation;
  - optimal ordering.

V-BLAST performance and multiplexing gain

- The multiplexing gain ζ_{M} quantifies the data rate gain:

$$\zeta_{\mathrm{M}} \equiv \lim_{\gamma_{\mathrm{AV}} \to +\infty} \frac{R(\gamma_{\mathrm{AV}})}{\log \gamma_{\mathrm{AV}}} \leq \min\{M, N\}.$$

- V-BLAST attain the maximum multiplexing gain, i.e. $\zeta_{\mathrm{M}} = M$.
- Diversity gain of $\zeta_{\mathrm{D}} = N - M + 1$ (no transmit diversity).



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Diversity-multiplexing trade-off

- There is an optimal trade-off between diversity and multiplexing gain in a $M \times N$ MIMO system:

$$\zeta_{\mathrm{D}} = (M - \zeta_{\mathrm{M}} + 1)(N - \zeta_{\mathrm{M}} + 1).$$

- No multiplexing if $\zeta_{\mathrm{D}} = MN$. No transmit or receive diversity if $\zeta_{\mathrm{M}} = \min\{M, N\}$.
- Systems capable of providing both transmit diversity and multiplexing (in rank deficient scenarios) are a matter of interest.

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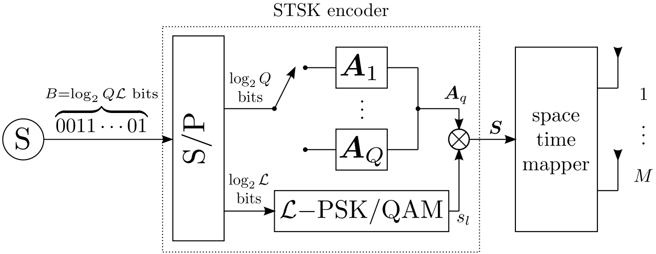
Space-Time Shift Keying



- Efficient modulation that employs as information carriers:
  - a symbol s_l taken from an $\mathcal{L}$-ary constellation;
  - a particular spreading of the selected symbol over the M transmit antennas and T symbol intervals.
- Q different ways of spreading are possible.
- Each way of spreading is represented by a dispersion matrix (DM) $\boldsymbol{A}_q \in \mathbb{C}^{M \times T}$.
- A DMs set is the set of the Q possible spreadings $\{\boldsymbol{A}_q\}_{q=1}^{Q}$.

What *spreading a symbol* means - I

- The space-time codeword transmitted by a STSK scheme is a symbol s_l spread in time and space according to $\boldsymbol{A}_q$.
- Example: $M = 2$, $T = 2$, $Q = 2$, BPSK ($\mathcal{L} = 2$) and:

$$\{\boldsymbol{A}_q\}_{q=1}^{2} = \{\boldsymbol{A}_1, \boldsymbol{A}_2\} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}.$$

- Mapping: $\log_2 \mathcal{L}Q = \log_2 4 = 2$ bits determine a codeword.
  - 1^{st} bit select the symbol: $0 \rightarrow s_1$, $1 \rightarrow s_2$;
  - 2^{nd} bit select the DM set: $0 \rightarrow \boldsymbol{A}_1$, $1 \rightarrow \boldsymbol{A}_2$;
- E.g. $11\ 10$ corresponds to $\boldsymbol{S} = s_2\boldsymbol{A}_2$ and $\boldsymbol{S} = s_2\boldsymbol{A}_1$, i.e.:

	11		10	
	slot 1	slot 2	slot 1	slot 2
antenna 1	0	s_2	s_2	0
antenna 2	s_2	0	0	s_2
	$\underbrace{\qquad\qquad}_{\boldsymbol{S}=s_2\boldsymbol{A}_2}$		$\underbrace{\qquad\qquad}_{\boldsymbol{S}=s_2\boldsymbol{A}_1}$	

What *spreading a symbol* means - II

- Another example: $M = 2, T = 2, Q = 2$, BPSK ($\mathcal{L} = 2$) and:

$$\{\boldsymbol{A}_q\}_{q=1}^{2} = \{\boldsymbol{A}_1, \boldsymbol{A}_2\} = \left\{ \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}, \frac{1}{\sqrt{5}} \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} \right\}.$$

- Mapping: $\log_2 \mathcal{L}Q = \log_2 4 = 2$ bits determine a codeword.
  - 1^{st} bit select the symbol: $0 \rightarrow s_1$, $1 \rightarrow s_2$;
  - 2^{nd} bit select the DM set: $0 \rightarrow \boldsymbol{A}_1$, $1 \rightarrow \boldsymbol{A}_2$;
- E.g. $01\ 00$ corresponds to $\boldsymbol{S} = s_1\boldsymbol{A}_2$ and $\boldsymbol{S} = s_1\boldsymbol{A}_1$, i.e.:

	01		00	
	slot 1	slot 2	slot 1	slot 2
antenna 1	$-s_1 \frac{1}{\sqrt{5}}$	$s_1 \frac{2}{\sqrt{5}}$	$s_1 \frac{1}{\sqrt{5}}$	$s_1 \frac{2}{\sqrt{5}}$
antenna 2	$s_1 \frac{2}{\sqrt{5}}$	$s_1 \frac{1}{\sqrt{5}}$	$s_1 \frac{2}{\sqrt{5}}$	$-s_1 \frac{1}{\sqrt{5}}$
	$\underbrace{\qquad\qquad\qquad}_{\boldsymbol{S}=s_1\boldsymbol{A}_2}$		$\underbrace{\qquad\qquad\qquad}_{\boldsymbol{S}=s_1\boldsymbol{A}_1}$	

Properties of STSK

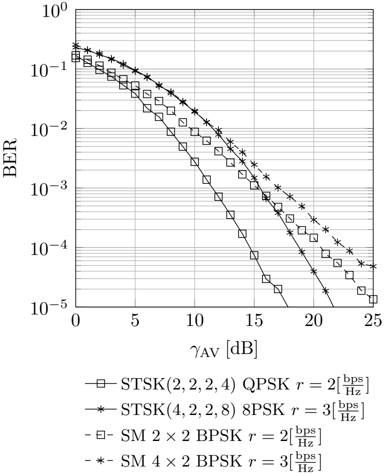
- Simple encoding and decoding.
- $\zeta_{\mathrm{D}} = N \min\{M, T\}$ and $\zeta_{\mathsf{M}} = 1$ (no multiplexing gain, but efficiently increase the modulation order).
- The set of the DMs $\{\boldsymbol{A}_q\}_{q=1}^{Q}$ has to be accurately chosen for ensuring a good performance.
- STSK provides a system rate of:

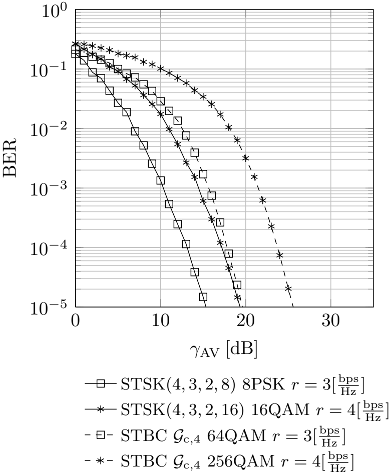
$$R_{\mathrm{STSK}} = \frac{\log_2 \mathcal{L}Q}{T} \quad \left[\frac{\text{bits}}{\text{channel use}}\right].$$

- The parameter T (number of slots on which a symbol is spread) permits to trade-off diversity and data rate.

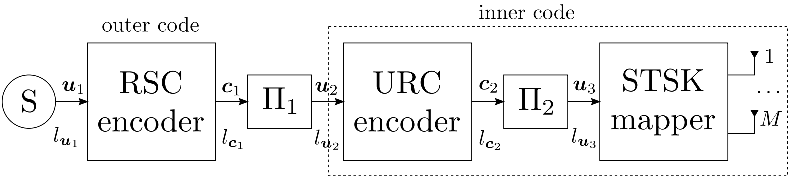
Performance of STSK

- Better performance than Spatial Modulation (SM) and STBC.





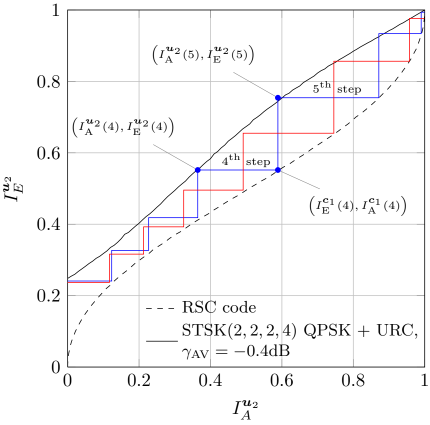
Turbo STSK scheme

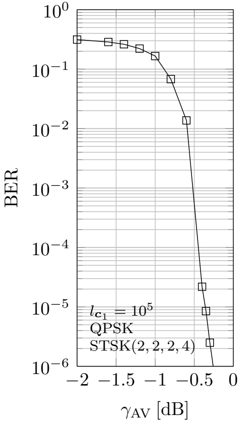


- A three stage serially concatenated turbo STSK scheme:
  - near-capacity performance;
  - exploits the diversity/multiplexing trade-off offered by the STSK architecture.
- A Recursive Systematic Convolutional (RSC) and an Unitary Rate Convolutional (URC) encoder are introduced before the STSK encoder.

EXIT chart analysis of the turbo STSK scheme

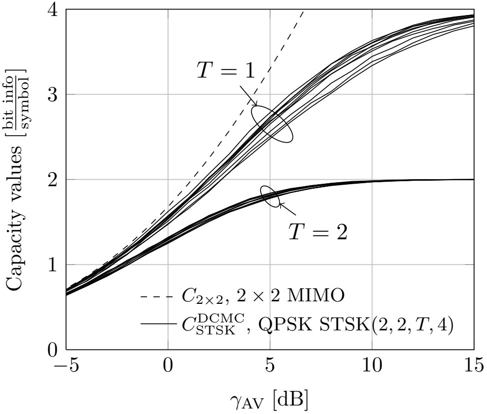
- EXIT charts study the convergence behaviour of a concatenated code avoiding Monte Carlo simulations.
- Easily calculate the decoding threshold Γ_{th}, i.e. the lowest SNR supporting an error free decoding.

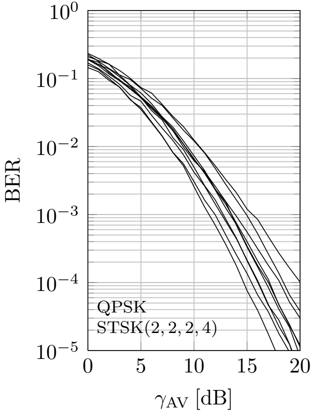




How DMs affect STSK performance

- Capacity and BER performance of the STSK scheme crucially affected by the DMs set $\{\boldsymbol{A}_q\}_{q=1}^{Q}$.





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  - Previously proposed design criteria
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Previous work

- Performance Evaluation:
  - The "quality" of a DMs set $\{\boldsymbol{A}_q^{(i)}\}_{q=1}^{Q}$ is evaluated through a non-negative fitness function $\mathcal{F}_{(\cdot)}$:

$$\mathcal{F}_{i,(\cdot)} = \mathcal{F}_{(\cdot)} \left(\{\boldsymbol{A}_q^{(i)}\}_{q=1}^{Q} \right) \geq 0.$$

  - The FF depends on the adopted evaluation criterion:
    - rank-and-determinant criterion;
    - capacity criterion.
  - Both do not target directly the performance of the turbo STSK scheme at low SNRs.
- Search for good DMs:
  - the DMs set with the best fitness among N_{search} randomly generated candidates was chosen.

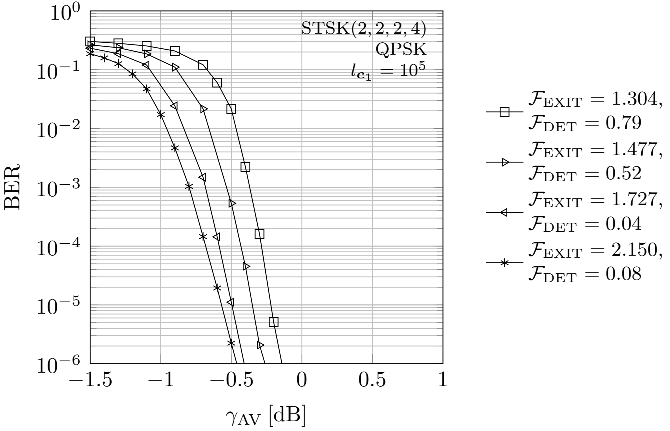
Outline

The novel design criterion based on EXIT chart

- Γ_{th} is calculated set with a bisection inspired method.
- The FF is defined as $\mathcal{F}_{\mathrm{EXIT}} = \exp(-\Gamma_{\mathrm{th}})$.
- The novel criterion:
  - selects the DMs sets having the lowest decoding threshold;
  - directly targets the convergence behaviour of the particular turbo coded STSK scheme;
  - outperforms previously proposed criterion that did not target directly the performance of the coded scheme at low SNRs.

Results using the EXIT criterion

- The novel criterion outperforms previously proposed criteria.
- Drawback: high computational complexity.



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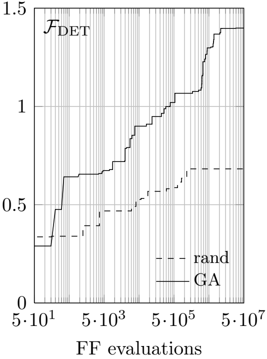
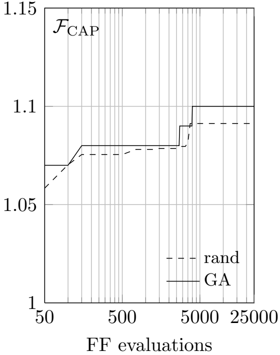
The reasons to adopt a GA aided search

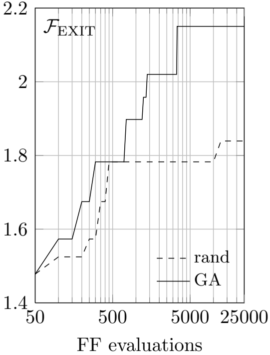
- Random selection:
  - high number of candidates to be evaluated for finding a satisfying DMs set (i.e. high number of FF evaluations);
  - does not guarantee the convergence to a target solution.
- Genetic Algorithms (GAs) combine:
  - wide exploration of the search space (typical of random selection);
  - convergence to a target solution (typical of numeric maximisation algorithms).
- Hence, GAs are suited for maximisation problems having:
  - wide search space;
  - objective functions not well understood and unavailable in closed form.
- A properly designed GA was applied to the search of DMs sets for STSK systems.

Computational complexity: GA vs random

- Evaluated system: $\mathrm{QPSK\ STSK}(2,2,2,4)$, $l_{\boldsymbol{c}_1} = 10^5$.
- GA matches the result of a random search using less FF evaluations.
- GA always obtains better results than a random search with the same number of FF evaluations.

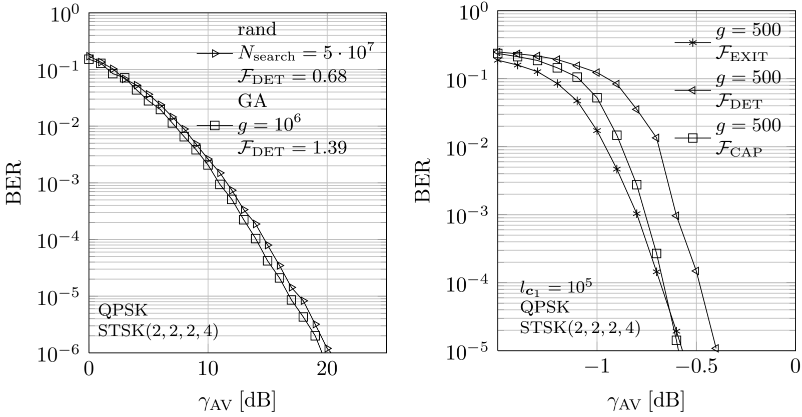






Performance improvement

- BER improvement in the uncoded scenario combining the GA aided search and the DET criterion.
- BER improvement in the coded scenario combining the GA aided search and the EXIT criterion.



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Frequency selective fading channels

- Fading may become frequency selective in high speed transmissions.
- $W >> B_\mathrm{c}$: this causes ISI, hence a degraded performance.
- A frequency selective fading channel model:

$$h[n] = \sum_{l=0}^{L_\mathrm{ch}-1} h[l]\delta[n-l].$$

- $h[l]$ are RVs that depend on the channel model.
- MIMO frequency selective fading channel model:

$$\boldsymbol{H}[n] = \sum_{l=0}^{L_\mathrm{ch}-1} \boldsymbol{H}[l]\delta[n-l] \in \mathbb{C}^{N\times M}.$$

- Every matrix $\boldsymbol{H}[l]$ has uncorrelated entries if the antenna elements are sufficiently spaced.

Orthogonal Frequency Division Multiplexing

- Transmits symbols on N_c parallel sub-carriers having a sufficiently low bit rate to avoid dispersion.
- A single dispersive channel ($W \propto \frac{1}{\mathcal{T}}$) is transformed in N_c parallel flat fading sub-channels ($W_s \propto \frac{1}{\mathcal{T}_s} = \frac{1}{\mathcal{T} N_\mathrm{c}}$), maintaining a constant overall bit rate.
- The time domains samples of the OFDM signal can be obtained through simple signal processing operations:

$$s(t) = \sum_{k=0}^{N_\mathrm{c}-1} S[k] e^{j2\pi k \Delta f t}.$$

Assuming $\Delta f = \frac{1}{\mathcal{T}_s} = \frac{1}{\mathcal{T} N_\mathrm{c}}$ one obtains:

$$\begin{aligned} s[n] = s(t = n\mathcal{T}) &= \sum_{k=0}^{N_\mathrm{c}-1} S[k] e^{j\frac{2\pi}{N_\mathrm{c}}kn} \\ &= N_\mathrm{c} \cdot \mathrm{IDFT}\{S[k]\}. \end{aligned}$$

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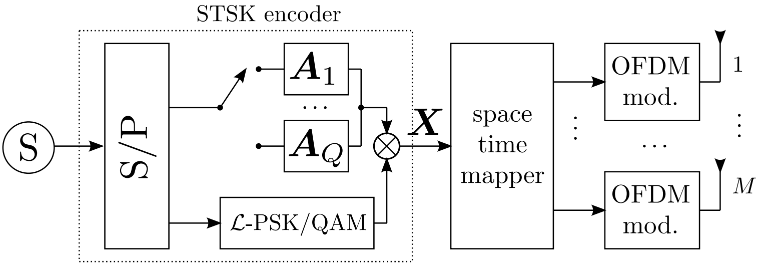
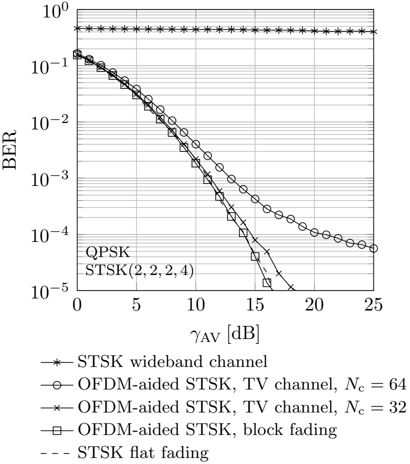
The novel OFDM-aided STSK scheme

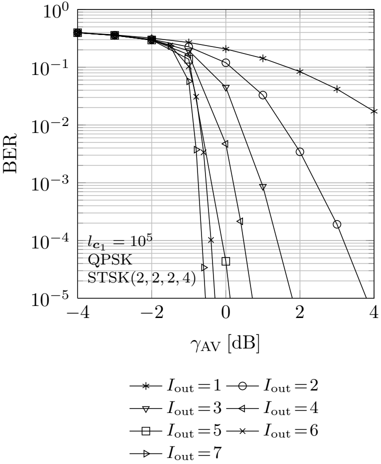


- Same encoding as classical STSK scheme.
- Multiple STSK codewords $X = A_q s_l$ are mapped to OFDM frames according to a specific mapping rule.

Performance of the OFDM-aided STSK scheme

- The proposed system approaches the performance obtained by the STSK scheme in the flat fading channel in both the uncoded and coded scenarios.





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What this thesis achieved

- New criterion based on EXIT charts for directly targeting the low SNR performance of turbo STSK schemes.
- Novel GA for reducing computational complexity of the DMs search whilst obtaining better performance.
- Novel OFDM-aided STSK scheme for dispersive channels.

- Results published in:
  - F. Babich, A. Crismani, M. Driusso, and L. Hanzo, "Design Criteria and Genetic Algorithm Aided Optimization of Three-Stage-Concatenated Space-Time Shift Keying Systems," *IEEE Signal Processing Letters*, vol. 19, pp. 543 - 546, August 2012;
  - M. Driusso, M. I. Kadir, F. Babich, and L. Hanzo, "OFDM Aided Space-Time Shift Keying for Dispersive Downlink Channels," in *IEEE Vehicular Technology Conference*, September 2012.

Ideas for future works

- Finely tuning the GA (e.g. discrete encoding).
- Reduced complexity STSK decoding algorithms for reducing the complexity of the EXIT criterion.
- New kind of mappings for the OFDM-aided STSK scheme.
- OFDM-aided Space-Time-Frequency Shift Keying (STFSK) scheme employing 3D DMs.

Thanks for your attention.

The MIMO channel model

- According to the Clarke's 2D isotropic scattering model:
  - 2D propagation environment (long path length compared to antennas heights);
  - waves arrives to receive antennas from all directions with equal probabilities;
  - isotropic receive and transmit antennas.
- This leads to a flat fading channel model where $h \sim \mathcal{CN}(0, 1)$ (Rayleigh flat fading).

- The $M \times N$ MIMO channel model adopted:
  - assumes spaced transmit and receive antennas: $d >> \frac{\lambda}{2}$;
  - can be modeled with a matrix $\boldsymbol{H} \in \mathbb{C}^{N \times M}$ having uncorrelated entries $h_{i,j} \sim \mathcal{CN}(0, 1)$.

DCMC Capacity of STSK

- Vectorial model when $\boldsymbol{S}_{l,q} = \boldsymbol{A}_q s_l$ is transmitted:

$$\boldsymbol{Y} = \boldsymbol{H}\boldsymbol{S}_{l,q} + \boldsymbol{Z} \in \mathbb{C}^{N\times T} \xrightarrow{\mathrm{vec}(\cdot)} \bar{\boldsymbol{Y}} = \bar{\boldsymbol{H}}\boldsymbol{\chi}\boldsymbol{K}_{l,q} + \bar{\boldsymbol{Z}} \in \mathbb{C}^{NT}$$

  where $\bar{\boldsymbol{H}} = I_T \otimes \boldsymbol{H} \in \mathbb{C}^{NT\times MT}$ and:

$$\boldsymbol{\chi} = \left[\mathrm{vec}\left(\boldsymbol{A}_1\right) \cdots \mathrm{vec}\left(\boldsymbol{A}_Q\right)\right] \in \mathbb{C}^{MT\times Q},$$

$$\boldsymbol{K}_{l,q} = [\underbrace{0\ \ldots\ 0}_{q-1}\ s_l\ \underbrace{0\ \ldots\ 0}_{Q-q}]^{\mathrm{T}} \in \mathbb{C}^{Q}.$$

- ML decoding rule:

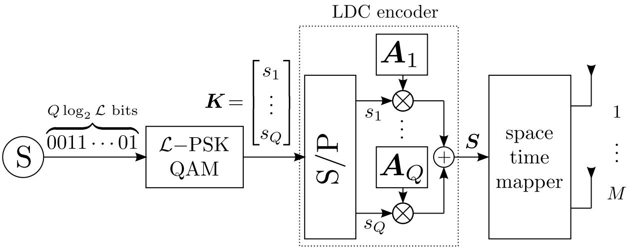
$$(\hat{l}, \hat{q}) = \arg\min_{l,q} \left\{ \left\| \bar{\boldsymbol{Y}} - \bar{\boldsymbol{H}}\boldsymbol{\chi}\boldsymbol{K}_{l,q} \right\|^2 \right\};$$

  which determines $\mathcal{L}Q$ codewords to be evaluated.

- DCMC capacity can be calculated as:

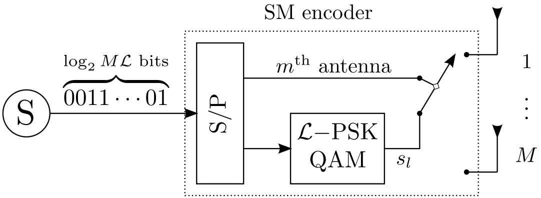
$$C^{\text{DCMC}}_{\text{STSK}} = \frac{1}{T} \max_{P(\boldsymbol{K})} \left\{ \mathbb{E}\left[\log_2 \frac{p(\boldsymbol{K}, \bar{\boldsymbol{Y}})}{P(\boldsymbol{K}) p(\bar{\boldsymbol{Y}})} \right] \right\} \quad \left[\frac{\text{bit info}}{\text{symbol}}\right].$$

Linear Dispersion Codes



- High degree of freedom architecture also for rank deficient scenarios: ζ_D and ζ_M can be chosen by the system designer.
- Q symbols weight Q carefully designed dispersion matrices $\boldsymbol{A}_q \in \mathbb{C}^{M\times T}, \forall q$.
- $\zeta_\mathrm{D} = N \min\{T, M\}$ (full receive diversity and full transmit diversity if $T \geq M$).
- $\zeta_\mathrm{M} = \frac{Q}{T}$ and $R_\mathrm{LDC} = \frac{Q}{T} \log_2 \mathcal{L}$ bits per channel use.
- High encoding and decoding complexity ($\mathcal{L}^Q$ codewords).

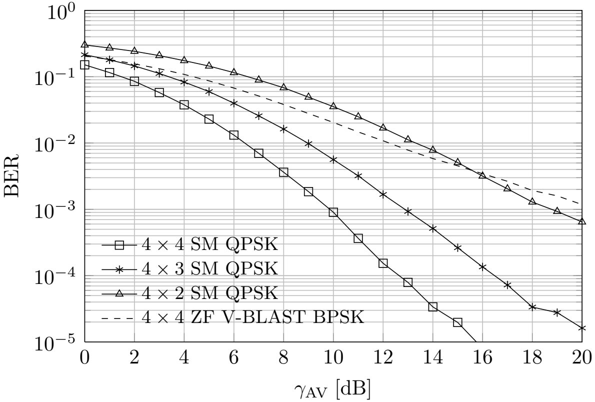
Spatial Modulation



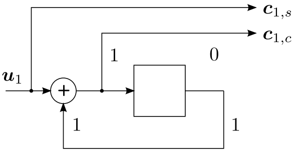
- A symbol and an antenna index are used as information carriers.
- Data rate of $R_{\text{SM}} = \log_2 \mathcal{L}M$ bits per channel use.
- Simple encoding and decoding.
- Space Shift Keying (SSK) is a particular case of SM.

Performance of SM

- No transmit diversity gain. Full receive diversity gain ($\zeta_{\mathrm{D}} = N$).
- No multiplexing gain. SM efficiently increase the order of the modulation.
- Better BER performance than V-BLAST even if $N < M$.



More on the turbo STSK scheme



- RSC encoder: rate $R_{c,1} = 1/2$, constrain length $\mathcal{K}_1 = 2$ (i.e. memory $\nu_1 = 1$), generator polynomial $p_{g,1} = (2)_8$ and feedback polynomial $p_{f,1} = (3)_8$.
- URC encoder: same as RSC, but the systematic bit is punctured.
- SISO STSK demapper:

$$L_e\left(x_b\right)=\log\left\{\frac{\sum_{\boldsymbol{K}_j\in\mathbb{K}_0^b}\exp\left[-\gamma_{\mathsf{AV}}\left\|\bar{\boldsymbol{Y}}-\bar{\boldsymbol{H}}\boldsymbol{\chi}\boldsymbol{K}_j\right\|^2-\sum_{n=1,n\neq b}^{B}x_n^{\boldsymbol{K}_j}L_a\left(x_n\right)\right]}{\sum_{\boldsymbol{K}_j\in\mathbb{K}_1^b}\exp\left[-\gamma_{\mathsf{AV}}\left\|\bar{\boldsymbol{Y}}-\bar{\boldsymbol{H}}\boldsymbol{\chi}\boldsymbol{K}_j\right\|^2-\sum_{n=1,n\neq b}^{B}x_n^{\boldsymbol{K}_j}L_a\left(x_n\right)\right]}\right\}.$$

EXIT chart analysis of the turbo STSK scheme

- Analises the convergence behaviour of a turbo coded scheme at low SNRs (in the waterfall region):
  - by visualising the information exchange between constituent codes of the concatenated scheme.
- Measures the mutual information $I(\boldsymbol{x}, L(\boldsymbol{x}))$ between bits $\boldsymbol{x}$ and LLRs $L(\boldsymbol{x})$.
- An EXIT curve of a constituent code visualises $I_{\mathrm{E}} = I_{\mathrm{e}}(\boldsymbol{x}, L_{\mathrm{e}}(\boldsymbol{x}))$ as a function of $I_{\mathrm{A}} = I_{\mathrm{a}}(\boldsymbol{x}, L_{\mathrm{a}}(\boldsymbol{x}))$.
- The EXIT chart jointly plots the EXIT curves of the constituent codes.
- In serially concatenated schemes, the curve on the inner code depends on γ_{AV}.

Previously proposed evaluation criteria

- Rank-and-determinant criterion:
  - The pairwise error probability between the codewords $\boldsymbol{S}$ and $\boldsymbol{S}'$ can be expressed as:

$$P\left(\boldsymbol{S} \to \boldsymbol{S}'\right) \leq (\Delta)^{-N} \left(\frac{\gamma_{\mathsf{AV}}}{4M}\right)^{-rN} .$$

    where r and Δ depends on $\boldsymbol{S}_\Delta = (\boldsymbol{S} - \boldsymbol{S}')(\boldsymbol{S} - \boldsymbol{S}')^{\mathrm{H}}$.
  - This criterion aims to minimise the worst $P\left(\boldsymbol{S} \to \boldsymbol{S}'\right)$ by maximising both:

$$\mathcal{F}_{\mathrm{RANK}} = r_{\min} = \min_{\boldsymbol{S},\boldsymbol{S}'} \{r\} \leq \min\{M, T\},$$

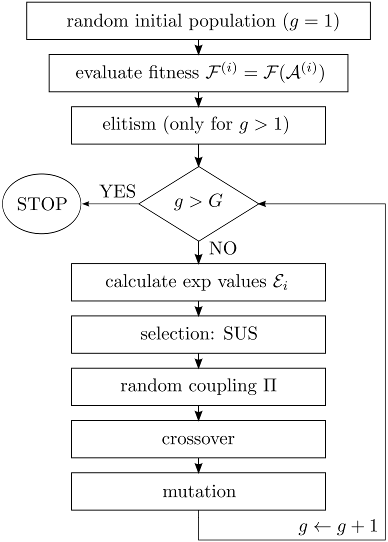
$$\mathcal{F}_{\mathrm{DET}} = \Delta_{\min} = \min_{\boldsymbol{S},\boldsymbol{S}'} \{\Delta\} \geq 0.$$

- Capacity criterion:
  - The capacity of a STSK scheme is a function of $\{\boldsymbol{A}_q\}_{q=1}^{Q}$.
  - This criterion aims to select the DMs set that maximise the value $\mathcal{F}_{\mathrm{CAP}} = C_{\mathrm{STSK}}^{\mathrm{DCMC}}$ for a fixed SNR value.

What is a GA?

- Algorithm that applies the concepts of natural evolution to computational problems such as maximisation.
- Keywords:
  - *population* $\rightarrow$ the set of N_{pop} individuals $\{\mathcal{A}^{(i)}\}_{i=1}^{N_{\text{pop}}}$
  - *individuals* $\rightarrow$ every candidate solution $\mathcal{A}^{(i)} = \{\boldsymbol{A}_q^{(i)}\}_{q=1}^{Q}$
  - *genes* $\rightarrow$ $\boldsymbol{a}_{q,t}^{(i)} \in \mathbb{C}^M$ (continue encoding)
  - *generation* $\rightarrow$ the population in a particular step of the GA
  - *parents* $\rightarrow$ individuals selected from the population, that will be transformed for constituting the new generation
  - *children* $\rightarrow$ individuals constituting the new generation
  - *crossover* $\rightarrow$ generates two children by randomly swapping the genes of a couple of parents
  - *mutation* $\rightarrow$ randomly changes the gene of an individual
  - *elitism* $\rightarrow$ replaces the least fit children with the fittest parents

The developed GA



random initial population ($g = 1$)

evaluate fitness $\mathcal{F}^{(i)} = \mathcal{F}(\mathcal{A}^{(i)})$

elitism (only for $g > 1$)

$g > G$

YES

STOP

NO

calculate exp values $\mathcal{E}_i$

selection: SUS

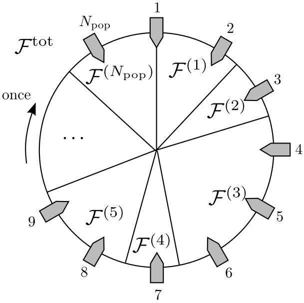
random coupling Π

crossover

mutation

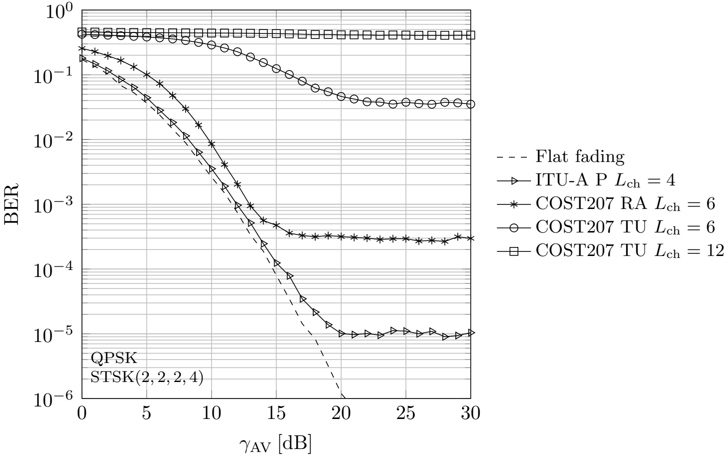
$g \leftarrow g + 1$

- Stochastic Universal Sampling (SUS) is a fitness proportional selection process responsible of the selection of the N_{pop} individuals constituting the set of the parents.

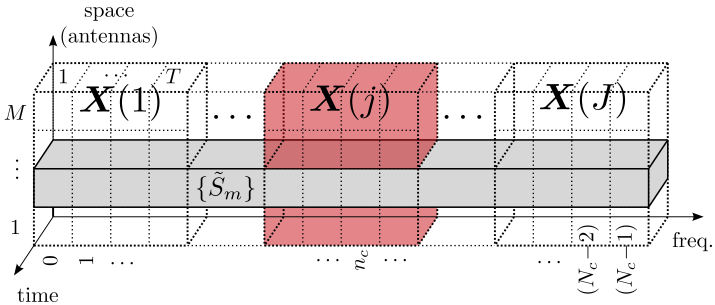


STSK in wideband channels

- ISI causes a degraded performance.

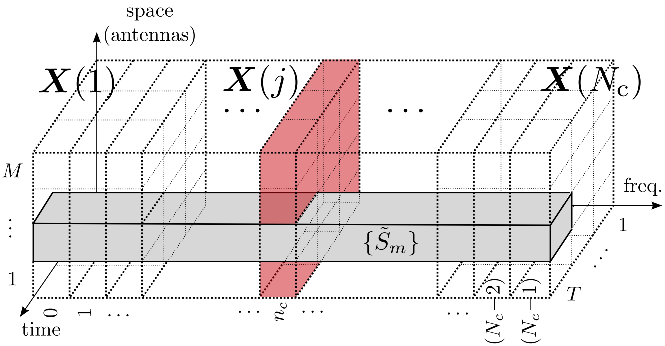


OFDM aided STSK mapping 1



- The columns of $J = N_c/T$ STSK codeword are transmitted in same OFDM frame:
  - mapping the columns of a STSK codeword to T adjacent sub-carriers.
- N_c has to be multiple of T.
- Well suited for fast fading channels, since the STSK codewords are not dispersed across the time dimension.

OFDM aided STSK mapping 2



- $J = N_{\mathrm{c}}$ STSK codewords are transmitted using T successive OFDM frames:
    - transmitting a single STSK codeword on one sub-carrier along T adjacent OFDM frames.
- A delay of $(T-1)\mathcal{T}N_{\mathrm{c}}$ is introduced at the receiver for decoding an OFDM frame.
- Well suited for slow fading channels.