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On a double aspect of natural numbers as abstract objects and universals

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ABSTRACT: Arithmetical expressions behave either as singular terms in identity statements or as predicates in adjectival statements. Those forms of syntactical behaviour often give rise to opponent ontological accounts of natural numbers: the *substantival* use of arithmetical expressions supports the interpretation of numbers as abstract objects whereas the *predicative* use can support an account of them as universals. The paper suggests that the two forms should be regarded as equivalent to each other. Then it defends the view that the double linguistic behaviour of natural numbers is indicative of a double metaphysical status of them as both objects and universals.

Key words: singular term, predicate, number, object, universal

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1. The dilemma

Arithmetical expressions appear to behave syntactically in two different ways, either as singular terms in identity statements or as predicates of concepts in adjectival statements. To take an example from Frege's *Grundlagen* §57, "the number of Jupiter's moons is 4" (1) is an identity statement where the arithmetical expressions possess places of singular terms. The verb "is" is not a copula here, but it means: *is identical to*. A different way to write down (1) is: "Jupiter has 4 moons" or "There are 4 moons of Jupiter" (2) where the arithmetical expression "4" behaves as an adjective. However, Frege defended the view that every arithmetical statement of the form (2) should be reparsed as (1).

The two forms of syntactical behaviour of arithmetical terms frequently give rise to different accounts of the ontological status of natural numbers. The form of the identity statement (1) is "$Nx : Fx \equiv n$" ("*The number of Fs is identical to n*"). I will call it the *substantival* form because it supports realistic accounts of natural numbers. In particular, it supports the Fregean view that natural numbers are objects and that arithmetical terms are their names. By contrast, the form of the adjectival statement (2) is "$\exists_n xFx$" ("*There are exactly n Fs* (*i.e. n items that fall under the concept F*)"). I will call it the *predicative* form since it emphasizes the predicative function of arithmetical expressions. So, a dilemma arises for those philosophers who take under consideration linguistic issues in order to support their metaphysical claims. Crispin Wright has described exactly this dilemma in the following passage:

> "…the choices are just two: either (following Frege) we take the basic form of numerical expression to be a singular term, definable by reference to an operator on concepts and re-parse ordinary adjectival statements of number as statements of identity; Or we take the basic form of numerical expression to be a predicate of concepts, i.e. a quantifier, and seek to re-parse the apparently substantival uses of numerical vocabulary with which number theory abounds." (Wright 1983, p.36)

ON A DOUBLE ASPECT OF NATURAL NUMBERS AS ABSTRACT OBJECTS AND UNIVERSALS

This paper takes in account both forms of numerical expressions and their respective metaphysical interpretations. It discusses the difficulties of the standard interpretations of the two forms and it investigates the relation that holds between them. Then it articulates a proposal according to which the substantival form is equivalent to the predicative form by presenting an appropriate equivalence principle. It concludes with suggestions about a possible account of natural numbers as entities with a double linguistic appearance in arithmetical language. It asserts that this double linguistic appearance is suggestive of the double ontological status of them.

2. The substantival use of arithmetical expressions

According to Neofregeanism, the individual instances of the concept *natural number*, namely the numbers 0, 1, 2, 3,… are taken to be abstract objects (abstract particulars). In *Grundlagen* §57, Frege suggested that the adjectival sentence "Jupiter has 4 moons" should be re-parsed as the identity statement "The number of Jupiter's moons is 4" so that the arithmetical expressions cease to play an adjectival role and behave like proper names. Frege believed that this form of arithmetical statements reveals the ontological status of natural numbers as objects. Hence, the Neo-Fregeans (Wright 1983), (Hale and Wright 2001) appeal to the substantival form "$Nx : Fx \equiv n$" *("the number of Fs is identical to n")* and they defend Frege's account. They claim that a proper argument in favour of mathematical realism can be based on it: (a) arithmetical expressions function in certain atomic sentences as singular terms (b) the sentences in which they function so are true. So, there are objects to which those expressions refer. On the basis of the neo-Fregean argument, natural numbers are taken to be the referents of singular arithmetical terms which occur in appropriate true sentences. Thus, the problem about the status of natural numbers is reduced to questions concerning the truth of certain arithmetical identities and the syntactical function of arithmetical expressions occurring in them.

A basic presupposition of the Neo-Fregean account is that we can fix truth conditions for arithmetical identities of the form "$Nx : Fx \equiv Nx : Gx$" ("*the number of Fs is identical to the number* *of Gs*”). This requirement is met by means of Hume’s Principle that is stated by Frege himself in *Grundlagen* §63, 68:

$$(N\equiv) \qquad (\forall F)(\forall G)[(Nx:Fx \equiv Nx:Gx) \leftrightarrow (F1-1G)]$$

(the number of *Fs* is identical to the number of *Gs* if and only if *Fs* are 1-1 correlated to *Gs).* By means of this 2nd order quantified equivalence, we can recognize a number *as the same object* every time we meet it in arithmetical contexts. Hence, Hume’s Principle is considered as a criterion of identity for natural numbers.

On the Neo-Fregean account, an object is a possible or actual referent of a singular term. So one of their concerns is to settle certain criteria by which we can discriminate singular terms from other types of expressions and then assert that arithmetical expressions are singular terms. Frege himself remarked that arithmetical terms usually occur in identity statements, they come after the definite article “the” and they get positions of the logical subjects of arithmetical sentences, so they behave like proper names. Hale and Wright, however, attempted to formulate strict syntactical criteria which aim to discriminate singular terms from other expressions. This point is important for Neo-Fregeanism, since they make use of the syntactical priority thesis according to which one must first certify the syntactical status of a range of expressions as singular terms and afterwards make semantic claims about their ability to refer. The Neo-Fregeans’ criteria for singular termhood include: Dummett’s inferential criteria (1973, pp. 59-60), the negation asymmetry criterion and some extra supplementary tests (Hale and Wright 2001, pp. 31- 47, 49-70).

To cut a long story short, the Neo-Fregean proposal aims to rule out quantifier words, indefinite noun phrases, predicates and relational expressions as well as other phrases which express generality but they often stand in places where singular terms can go. So, they claim that singular terms can be recognized among several kinds of expressions. Besides, since arithmetical expressions pass successfully the alleged syntactical criteria they must be characterized as singular terms. Hence, on their view, singular termhood can be secured for arithmetical expressions. Yet, this point has been controversial.

Because of the difficulties concerning the characterization of arithmetical terms as singular terms, the Neo-Fregean defence of natural numbers as objects has received much criticism. The degree of efficiency of the proposed syntactical criteria has been questioned.

They have been taken not to hit their target, i.e. they do not succeed in excluding plenty of troublesome and misleading expressions. For example, Szabó correctly notes that expressions like “the existence of a proof”, “the occurrence of a symptom”, “the average income”, etc. cannot be excluded by means of the proposed Neo-Fregean criteria (Loux and Zimmerman 2003, pp. 38-39). One can additionally mention plenty of expressions as, for example, “Mary’s smile”, “Arthur’s grimace”, “John’s glance” etc.

To be fair, it should rather be asserted that the Neo-Fregean syntactical criteria supply necessary but not sufficient conditions for a term to be a singular term. That is, if “t” *is* a singular term then it passes the proposed tests but not all expressions which pass successfully the tests are singular terms. So, it can be asserted that the syntactical discrimination of singular terms from other expressions is not testified in a satisfactory way. Perhaps, the criteria need further refinement as Bob Hale himself admitted: “what kind of assurance can we have that the battery of tests as it now stands accomplishes what it was designed to do –that further adjustments or even the addition of further tests will not be found necessary?” (Hale 2001, p.68) It seems that it is a very ambitious task to achieve a complete and sharp distinction between singular terms and other types of expressions by means of syntactical tests. However, this requirement yields a serious difficulty for the Neo-Fregean account which has not been coped with yet.

Since the syntactical priority thesis of the Neo-Fregean account and the very requirement of syntactical tests of singular termhood, involves us in the difficulties stated above, we might depart from it and endorse an alternative account. E. J. Lowe provides a different response to the question what an object is by objecting to all approaches that are more or less based on syntax (e.g. those approaches that take an object to be anything that can be referred to by a singular term or anything that can be the value of a variable of quantification). On Lowe’s account, the term “object” applies to any item that enjoys determinate identity-conditions. Lowe calls his account “*metaphysical*” in order to contrast it to the linguistic accounts that are based on the syntactical function of certain expressions. Hence, Lowe (2006, p.75) holds that objects are items for which determinate criteria of identity are supplied, and this is the characteristic that discriminates them from all other entities. It is true, however, that this account is very close to Frege's assertion that if a term stands for an object then this term should be associated with a criterion of identity. Moreover, it is quite obvious that arithmetical terms *are* associated with such a criterion, i.e. the criterion of identity set by Hume's Principle, so they should be classified as objects.

Nevertheless, there is an interesting corollary of Lowe's conception of an object. The conception of an object that is primarily based on satisfaction of certain identity conditions may allow for objects that are not particulars. Lowe (2006, p.77) explicitly notes that there may be entities that are universals but also meet identity conditions, so those entities should be taken to be objects as well. He takes, for example, natural *kinds* to be quite as much objects as particulars are. It is not the target of this paper to claim that numbers are *kinds* as Lowe maintains. However, the paper endorses the notion of "object" defended by Lowe that was presented above. On this notion, the claim that natural numbers are (abstract) objects does not necessarily mean that they are (abstract) particulars. So a natural number might be an abstract object, but not an abstract particular. This may not be an unpleasant result, though it clashes with the Neo-Fregean interpretation of natural numbers as abstract particulars. I will return to this issue later. The next section will discuss some basic interpretations of the predicative use of arithmetical expressions.

3. The predicative use of arithmetical expressions

The other way is to appeal to the predicative form "$\exists_n xFx$" *("There are exactly n Fs, i.e. n items that fall under F")* and -in contrast with Frege- maintain that there is no reason to reparse the statement "There are 4 moons of Jupiter" (2) as "The number of Jupiter's moons is 4" (1). For example, Musgrave (1986) holds that numerical quantification is a perfectly well understood notion on its own and numerical definite descriptions like "the number of Jupiter's moons" should be taken to be quantifiers instead of singular terms. Hodes (1984, pp.123-149) claims that even if numerals are taken to pass the syntactic tests for singular termhood, they not at all manage to effect reference to objects, since their own function is rather to encode cardinality quantifiers than referring to objects.

Arithmetical expressions can be regarded as quantifiers of sortal concepts:

$$\exists_0 \mathrm{xFx} \equiv \forall x \neg Fx$$

$$\exists_1 \mathrm{xFx} \equiv \exists x(Fx \wedge \forall yFy \rightarrow y = x)$$

………

$$\exists_{n+1} \mathrm{xFx} \equiv \exists x(Fx \wedge \exists_n y(Fy \wedge y \neq x))$$

A way to construe numerical quantification is to adopt a nominalistic account. Hence, the statement "There are two cats in our house" can be reformulated as "$\exists_2 xFx$" whereas the latter formulation can be paraphrased as (3)
"$\exists x \exists y(Fx \wedge Fy \wedge x \neq y \wedge (\forall zFz \rightarrow z \equiv x \vee z \equiv y))$"
(suppose *F* is the concept *cat in our house*). This example shows that quantifiers sustain a useful instrument for eliminativism. One of the main tasks of mathematical antirealism has been the re-formulation of scientific language so that the alleged arithmetical singular terms are eliminable. Thus, it is possible for the antirealist to argue that true scientific sentences are not ontologically committed to any mathematical entities. For example, (3) has no ontological commitments to any abstract objects. The target of the program Hartry Field has undertaken is to nominalise science, by demonstrating that numbers and other mathematical objects are dispensable. A mathematical nominalist might be an eliminativist so far as he tries to eliminate mathematical terms from the language where the more important scientific theories are written. Field's strategy was to divide scientific sentences into two components, from which the one is taken to be realistic (it has metaphysical commitments to abstract entities) whereas the other one is nominalised. A nominalist undertakes the task to prove that the nominalised component is the only one from the two components that is not eliminable. However, this turned to be a very ambitious task because, as Field admits (1989, pp. 235), the division of the mixed scientific language in mathematical and nominalised components, is not

always possible. Besides, even when it is possible, the realistic (mathematical) component resists elimination in a lot of cases.

If we do not endorse a nominalistic account of numerical quantifiers, we may construe them in a way that does not avoid metaphysical commitments, however those commitments may be of a very different sort from the realistic commitments under consideration in the previous section. Some philosophers who adopt the predicative use of arithmetical expressions have construed numbers as properties, e.g. Maddy (1990) holds that numbers are properties of sets and Yi (1999) has offered an account of natural numbers as collective properties of many objects together.

In his criticism to various accounts of numbers as properties of physical objects or natural collections, Frege remarked that an obvious difference between properties e.g. colours on the one hand and numbers on the other, is that a colour characterizes a physical stuff in a definite way though the way a number characterizes it depends upon the *special manner* we choose to consider that physical stuff. Hence, one could regard number 100 as the number that characterizes a bundle of canes but if we break every cane into two pieces then we can take number 200 to characterize the same physical mass etc. (*Grundlagen*, §23). That is, we firstly make use of a concept to consider the stuff that is before us. If, for example, our concept is *pack of game cards* then we have to apply number 1 to the relative stuff of cards but if our concept is *game card* then we have to apply number 52 to the same stuff. If our concept is molecule of game card then we will have to apply a different number etc. The point that has to be stressed here is that according to Frege, natural numbers should not be considered independently of some *concept F by which we view physical reality*. So Frege regarded numbers as *numbers of concepts*. He maintained that they are objects. After some efforts to define them by means of contextual definitions, he went on to define them by means of classes (extensionally).

However, here we will try to construe the predicative form of arithmetical expressions stated above, hence, a way to take in account arithmetical expressions as predicates of concepts is the following. Let F be the concept *cat in our house*. The sentence “the number of cats in our house is two” can be paraphrased to “there are two cats in our house” (“$\exists_2 xFx$”). The term “2” asserts how many instances of the sortal concept *F* there are. Then we may interpret numbers as properties of concepts. Suppose we take all those concepts that are "equinumerous" with the concept *F*, i.e. all the concepts, instances of which can be put in 1-1 correlation with the instances of the concept *F*. That means that all those concepts which are 1-1 correlated to *F* have *something* in common, i.e. they share a number. For example, the concepts: *parent of Mary*, *hand of George*, *eye of Jane* etc., namely, twice-instantiated concepts, are 1-1 correlated to the concept *F* because their instances can be put in 1-1 correspondence with the instances of *F*. Then *the number of the concept F* characterizes all those concepts which are 1-1 correlatable to *F*. The number 2, as a property, states the twice-instantiation of the above concepts[1].

Recall that Frege's final definition given in *Grundlagen*, §68, makes use of an extensional way to approach the situation: the number of the concept *F* is taken to be the "extension of the concept '*equinumerous with the concept F*'". Frege himself avoided systematically any interpretation of numbers as properties, since, as he notes in *Grundlagen* §57, each individual number shows itself for what it is, "a self-subsistent object". The alleged self-subsistence of natural numbers which "comes out at every turn" in arithmetical identities is for Frege a very important aspect of them, it is the basic characteristic for classifying them in the category of objects. His extensional reading previously mentioned matches to his account of numbers as objects, since extensions (i.e. classes) are usually taken to be objects.

However, the account that was suggested here, i.e. the interpretation of numbers as properties of concepts can possibly be compromised with the object-interpretation of numbers that was presented at the final part of the previous section. We will see how this goal can be achieved in the next two sections.

[1] For a distinction among *properties* and *concepts*, see Lowe (2006, pp. 85). He claims that properties are extralinguistic and extramental entities whereas concepts are not. He takes concepts to be ways of thinking or ways of intellectually grasping entities. Properties are ways of being, i.e. properties are ways entities are. Hence, *to possess a property* and *to possess a concept* is entirely different to each other.

4. The relation between the two forms of arithmetical expressions

This section raises the question how the two forms of arithmetical expression are related. We might apply a reduction strategy according to which the sub*stantival* form "$Nx : Fx \equiv n$" is reducible to the predicative form "$\exists_n xFx$". This option takes the alleged arithmetical singular terms to be reduced to numerical quantifiers. Moreover, if this reduction is construed as eliminativism then arithmetical singular terms are eliminative. To make use of an example mentioned above, the sentence "There are two cats in our house" can be reformulated as "$\exists_2 xFx$" (F is the concept *cat in our house*) whereas the latter formulation can be paraphrased as
"$\exists x \exists y (Fx \wedge Fy \wedge x \neq y \wedge (\forall z Fz \rightarrow z \equiv x \vee z \equiv y))$"
This strategy supports the view that arithmetical expressions are not genuine singular terms, so it undermines the semantic role of arithmetical terms as referring to objects. Those who adopt this nominalistic strategy regard the predicative form as the most fundamental of the two forms. That is, the sentence "there are 2 cats in our house" is taken to describe more fundamental facts than the sentence "the number of cats in our house is 2". The predicative form describes only concrete facts whereas the substantival form describes abstract and concrete facts. This reduction strategy asserts that the mixed facts described by the substantival form are reduced to the concrete facts that are described by the predicative form. Now, if the reverse reduction strategy is applied then the substantival form is taken to describe more fundamental facts than the predicative form. That is, the predicative form is reduced to the substantival ("there are 2 cats in our house" is reduced to "the number of the cats in our house is 2"). In contrast with nominalists, platonists may appeal to the latter reduction strategy since the concrete facts described by the predicative form are reduced to mixed (concrete and abstract) facts described by the substantival form. In other words, this strategy provides an inflationary reconstruction of the familiar numerical quantification. Besides, it takes the basic equipment of the world to consist of concrete *as well as* abstract facts.

Which of the two directions of the alleged reduction should be put forward? The answer is: none of the two. It should be remarked that in fact a reduction strategy is not conclusive in either of its two directions. The sentence “the number of cats in our house is 2” can be regarded as a paraphrase of the sentence “there are 2 cats in our house” or vice versa. It has correctly been remarked that paraphrases cannot be conclusive in either direction (Loux and Zimmerman 2003, p. 22).They often prove to be useful to both nominalists and platonists at their disputes, because realists put forward some sentences ontologically committed to abstract entities and nominalists return their nominalistic paraphrases back. Yet, one can interpret paraphrases either in an inflationary or a deflationary way to defend platonism or nominalism respectively. A similar objection to eliminating ontological commitments to abstracta by means of paraphrases is expressed by Alston (1958). He takes a sentence S to be apparently committed to some abstract entities and a sentence S´ to be a paraphrase of S so that S´ is not committed to those entities. This case is taken by nominalists to prove that S is only apparently committed to the alleged entities. Alston raises the question why S and not S´ is the sentence that deceives us? Could we not reverse the nominalistic argument in favour of platonism? Could we not say that, contrast to appearances, S´ is committed to the abstract entities described by S? Alston concludes that we do not have any means to discriminate those sentences that only apparently are committed to some entities from those sentences that really are committed to them. On the basis of the previous considerations, the conclusion that we have come across (in regard with the reduction strategy we applied above) is that neither of the two directions of the reduction is conclusive. So, among the two forms in question (the substantival and the predicative form) there is no prevalent to choose as a reduction basis.

Yet, an alternative option might be put forward. We can take the substantival form and the predicative form to be equivalent to each other. In this case, we will need to take in account the principle Nq: “$Nx : Fx \equiv n \leftrightarrow \exists_n xFx$” which is presented by Hale and Wright (2001, pp. 330-332). Wright offered a proof of this equivalence. Nq shows that for each Fregean number *n* it is established that *n* is the number of the concept *F* if and only if there are exactly *n* *Fs*. Nq can be regarded as no less than a material equivalence according to which the two sides have the same truth conditions and describe the same states of affairs. More precisely, the same states of affairs render the sentences appearing at the left and the right side of the equivalence, true. Independently of the reasons for which the Neo-Fregeans themselves are interested in Nq, my concern with Nq in this section is due to the fact that this principle appears to present the substantival and the predicative use of arithmetical expressions as equivalent to each other. It seems that numerical expressions behave equivalently both as singular terms and as predicates whereas this is exactly the focal point of this paper. That is, natural numbers are presented in arithmetical language in two different but equivalent ways. The substantival use of arithmetical expressions in "the number of Jupiter's moons is 4" and the predicative use of them in "there are 4 moons of Jupiter" are the two sides of the same coins.

5. A double metaphysical status?

Ramsey (1925) has questioned the very traditional distinction among objects and universals. Recall that he objected to the common assumption that there is some difference in kind between objects and universals and maintained that the subject-predicate distinction, even in contexts of atomic sentences, provides no sufficient grounds for a clear distinction between objects and universals. Hence, he favoured an eccentric claim that there is no real distinction between objects and universals. He took it to be obvious in case of "Socrates is wise" which can be turned round to the equivalent "wisdom is a characteristic of Socrates". He argued that those sentences assert the same fact and express the same proposition. So, he concluded that *which* linguistic use of the term "wisdom" we endorse (the substantival or the predicative) is *a matter either of literal style or of the point of view from which we approach the situation.*

A Ramseyan possible answer to our dilemma (among the *substantival* and the *predicative* form of arithmetical expressions) might also be that there is no real metaphysical distinction to be asserted on linguistic grounds (among numbers as objects and numbers as properties): which of the two forms (substantival or predicative) one chooses is a matter just of literal style or of the point of view from which she approaches the (arithmetical) fact.

However, such a solution will not be adopted here, since it might deflate the metaphysical problem concerning the status of natural numbers. This paper is interested in a more metaphysically loaded answer to the dilemma. This response will follow the basic assertion made in the previous section that natural numbers can be taken as (abstract) entities with two different (but equivalent) modes of linguistic presentation. Moreover, this assertion may have something more to say about the metaphysical status of natural numbers: namely, that numbers are both universals and objects.

Can there be any universals that are objects? Recall that in the second section, the account of object endorsed by Lowe was reminded. Lowe (2006) admits of entities which are both objects and universals. He says that there are entities which are universals but also satisfy identity conditions, so they should be taken to be objects as well. According to Lowe (2006, p.75), what discriminates objects from other entities is that they possess *determinate identity conditions*. As it has already been noted in the section 2, Hume's principle is an identity criterion for natural numbers since it sets truth conditions for arithmetical identity statements. So, we have to admit that, undoubtedly, natural numbers *are* objects, although they syntactically appear to behave in two different ways. Moreover, we saw that those ways are equivalent. Yet, according at least to Lowe's notion of 'object', objects are not necessarily particulars, so the case cannot be excluded that numbers are objects, being also universals.

On the other hand, some philosophers regard Frege's notion of an object quite broad. Ramsey has emphasized the point that the very Fregean distinction between complete and incomplete expressions which are in need of saturation cannot efficiently support any analogous distinction between complete and incomplete entities (e.g. objects and concepts). It has also been remarked that Frege's assumption that an expression that stands for an object does not contain empty places can apply as easily to concepts as to objects (Mellor and Oliver 1997, p.8). Hence, the expression "wisdom" can perfectly stand for an object since it contains no empty places. Thus, surprisingly, the supposed Fregean conception of an object can be taken to apply to concepts too. Mellor and Oliver (op.cit) note that this challenge arises because Frege's account of objects appears to be too broad.

So far, three points were emphasized: (1) that natural numbers are abstract entities that present themselves in language in two different ways, (2) natural numbers *are* objects since they possess determinate identity conditions (set by Hume's Principle), (3) the fact that numbers are objects cannot exclude the case that they may be another kind of entity as well. For example, they might be objects *and* universals, since Lowe's conception of an object allows for objects which are universals. A fourth assertion is needed to supplement the picture.

Recall the so called "*the paradox of 'the concept horse*'". Frege takes "the concept horse" to be a singular term whereas he takes "__ is a horse" to be a predicate. Hence, the crucial point is that "the concept horse" does not co-refer with "__ is a horse" because the former stands for an object while the latter stands for a concept. The so called paradox should rather be taken to be a situation which provides a double vision in metaphysics. If we are not willing to endorse a double vision approach then we will keep on considering it as a paradox. The point to be stressed is that there are Fregean objects corresponding to Fregean concepts. Now, let "wisdom" be a singular term whereas "__ is wise" be a predicate. Hence, "wisdom" does not co-refer with "__ is wise" because the former stands for an object while the latter stands for a property. As Wright has emphasized, the relation between a predicate and the correlated concept (or the correlated property), does not need to be construed as a relation between referring expressions and their referents. In his attempt to deal with the "'concept horse' paradox", Wright raised the question what form of semantic relation predication is. So Wright's assertion was that predication is not a species of reference (Hale and Wright 2001, pp. 86-88).

Wright investigated mainly two possible answers: a) that predicates stand for concepts (or properties) in the same way that singular terms stand for objects. b) the relation between a predicate and the associated entity is not that which obtains between a singular term and its referent. Wright rejected a) but he endorsed b). He claimed that predication, i.e. the relation between a predicate and the associated concept (or property) is not a species of reference. He suggested that this relation is better characterized by means of *ascription*. To see how *ascription* works, we have to remark that a predicate *ascribes* a concept (or a property) but it does not *refer* to that concept (or that property). Thus, the predicate "__ is wise" ascribes the property *wisdom.* (Or, the property *wisdom* is the *ascriptum* of the predicate "__ is wise" (not the referent)). So, the expressions "wisdom" and "__ is wise" differ precisely in that the former *refers to* wisdom whereas the latter *ascribes* wisdom.

Secondly, Wright made a further assumption that is important for the issue that is investigated in this paper. He suggested that the very entities which the predicates ascribe can also be the referents of singular terms. So the concept that is the *ascriptum* of the predicate "__ is a horse" can also be the referent of the Fregean singular term "the concept horse". Analogously, the property that is the *ascriptum* of the predicate "__ is wise" can also be the *referent* of the singular term "wisdom". Thus, the same entity may be both the *ascriptum* of a predicate and the *referent* of a singular term. This is our focal point. In other words, some entities are objects as the *referents* of certain Fregean singular terms *and* properties as the *ascripta* of predicates.

This suggestion has a very elucidating consequence for our dilemma, since we can treat numbers along the previous considerations. In this setting, wisdom *is an object* as the referent of the singular term "wisdom", *and a property too* as the *ascriptum* of the predicate "__ is wise". Similarly, it can be asserted that numbers are entities which are both the *ascripta* of certain predicates and the *referents* of the relative singular terms. It appears then that the very fact that arithmetical language offers a double aspect of linguistic behaviour of arithmetical expressions (as predicates and as singular terms) indicates also a double status at the metaphysical level. So, though we cannot always achieve strict syntactical distinctions, the role of language should be considered at least as suggestive.

In this paper, it was emphasized that arithmetical expressions behave in two ways in arithmetical language, namely as singular terms and as predicates. Those forms of syntactical behaviour give rise to metaphysical accounts of natural numbers either as objects or as properties, formulating thereby a dilemma for philosophers of arithmetic. This paper also took under consideration two basic accounts, one regarding each natural number as an object, the other taking each natural number to be a property. However, the aim was to show that the very fact of double linguistic presentation should indicate that natural numbers are entities with a double metaphysical status. So, we can now cope with our initial dilemma: which of the two is the number 4? Is it an object? Or is it a property? We saw that the arithmetical expression “4” behaves as a singular term in identity statements and as a predicate in adjectival statements. So our dilemma could be addressed by means of the suggestion that 4 is an abstract entity which is *both* an object and a property. It is an object to which a corresponding singular term refers. It is also a property which is the *ascriptum* of the corresponding predicate. Yet in this case, the two forms of linguistic behaviour of arithmetical expressions should be regarded as indicative of the Janus’ metaphysical face of natural numbers.

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